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五阶常微分方程的 Petrov-Galerkin 谱元法

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摘要： 通过区间剖分，降低数值逼近多项式的阶数，构造满足试探函数空间和检验函数空间的基函数，使得离散问题所对应的线性系统的系数矩阵是稀疏的，并可以进行有效地求解. 数值算例验证了五阶常微分方程的 Petrov-Galerkin 谱元法的有效性和高精度.

关键词： 五阶常微分方程；Petrov-Galerkin 谱元法；基函数；数值实验

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Petrov-Galerkin Spectral-Element Method for Solving Fifth-Order Ordinary Differential Equations

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Abstract: The polynomial order in the numerical approximation is reduced by partitioning the interval into several subintervals, and appropriate basis functions of the trial and test spaces are constructed. Which leads to a linear system with sparse coefficient matrix. Then, an efficient computational process is introduced to solve the linear system. Numerical experiment results demonstrate the high accuracy and effectiveness to the Petrov-Galerkin spectral-element method.

Keywords: fifth-order ordinary differential equation; Petrov-Galerkin spectral-element method; basis functions; numerical experiments

谱方法作为数值求解微分方程的方法之一，已被广泛应用于自然科学和工程技术问题的数值计算[1-7]. 近二十年来，奇数次高阶微分方程基于谱方法的数值模拟也受到不少科研工作者的关注[8-13]. 由于奇次高阶微分方程的高阶项缺少对称性，若使用配置点法求解奇次高阶微分方程边值问题，如果不能选取恰当的配置点或采用高阶多项式进行逼近，容易导致条件数比较高，造成计算不稳定[10]. 吴胜等[13]提出 Legendre-Petrov-Galerkin 谱元法对三阶微分方程进行求解. 由于将计算区间进行剖分，在每个区间上不需要使用太高的多项式阶数，将问题分解成一系列子问题后再用 Schur 补过程进行求解，计算稳定，且具有高精度. 本文研究五阶常微分方程的 Petrov-Galerkin 谱元逼近，主要考虑方程的数值计算.

1　Petrov-Galerkin 谱元法

记 $\Lambda=(-1,1)$，设 α 和 β 为正常数. 考虑五阶常微分齐次边值问题，即

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$$\left.\begin{aligned}&\alpha u+\beta u_{x,x,x}-u_{x,x,x,x,x}=f, \qquad x\in\Lambda,\\&u(\pm 1)=u_x(\pm 1)=u_{x,x}(1)=0.\end{aligned}\right\}\tag{1}$$

首先,将区间 Λ 剖分成 $K(K\geqslant 2)$ 个子区间

$$\Lambda_k=(a_{k-1},a_k), \qquad k=1,2,\cdots,K.$$

上式中:$-1=a_0<a_1<\cdots<a_K=1$. 记 $h_k=a_k-a_{k-1}$, $h=\max_{1\leqslant k\leqslant K}h_k$.

定义区间 Λ_k 到参考区间 Λ 的坐标变换为

$$\hat{x}_k\equiv\hat{x}_k(x)=\frac{2}{h_k}x-\frac{a_k+a_{k-1}}{h_k}, \qquad \forall x\in\Lambda_k.$$

定义分片多项式空间为

$$P_{N;K}(\Lambda):=\{u;u|_{\Lambda_k}\in P_N(\Lambda_k),k=1,2,\cdots,K\},$$

上式中:P_N 为 Λ_k 上次数不超过 N 的全体多项式组成的空间.

用 $\mathcal{N}$ 表示离散参数(N,K). 定义试探函数空间和检验函数空间为

$$V_{\mathcal{N}}=\{u;u|_{\Lambda}\in P_{N,K}(\Lambda);u\in C^1(\Lambda),u(\pm 1)=u_x(\pm 1)=u_{x,x}(1)=0\},$$

$$W_{\mathcal{N}}=\{u;u|_{\Lambda}\in P_{N+1,K}(\Lambda);u\in C^2(\Lambda),u(\pm 1)=u_x(\pm 1)=u_{x,x}(\pm 1)=0\}.$$

则问题(1)的 Petrov-Galerkin 谱元逼近形式为找 $u_{\mathcal{N}}\in V_{\mathcal{N}}$,使得

$$\alpha(u_{\mathcal{N}},v_{\mathcal{N}})-\beta(\partial_x^2u_{\mathcal{N}},\partial_xv_{\mathcal{N}})+(\partial_x^2u_{\mathcal{N}},\partial_x^3v_N)=(f,v_{\mathcal{N}}), \qquad \forall v_{\mathcal{N}}\in W_{\mathcal{N}}.\tag{2}$$

为了方便表达,将式(2)记为 $\alpha(u_{\mathcal{N}},v_{\mathcal{N}})=(f,v_{\mathcal{N}})$,记

$$\left.\begin{aligned}&\mathring{V}_N^k:=\{u^k;u^k\in P_N(\Lambda_k),u^k(a_{k-1})=u^k(a_k)=u_x^k(a_{k-1})=u_x^k(a_k)=u_{xx}^k(a_k)=0\},\\&\mathring{W}_N^k:=\{u^k;u^k\in P_{N+1}(\Lambda_k),u^k(a_{k-1})=u^k(a_k)=u_x^k(a_{k-1})=u_x^k(a_k)=\\&\qquad u_{x,x}^k(a_{k-1})=u_{x,x}^k(a_k)=0\}.\end{aligned}\right\}\tag{3}$$

定义

$$\varphi_j^k=\begin{cases}L_j(\hat{x}_k)-\dfrac{2j+3}{2j+7}L_{j+1}(\hat{x}_k)-\dfrac{2(2j+5)}{2j+7}L_{j+2}(\hat{x}_k)+\dfrac{2(2j+3)}{2j+9}L_{j+3}(\hat{x}_k)+\\ \qquad\dfrac{2j+3}{2j+7}L_{j+4}(\hat{x}_k)-\dfrac{(2j+3)(2j+5)}{(2j+7)(2j+9)}L_{j+5}(\hat{x}_k), & x\in\Lambda_k,\\ 0, & \text{其他}.\end{cases}$$

$$\phi_j^k=\begin{cases}L_j(\hat{x}_k)-\dfrac{3(2j+5)}{2j+9}L_{j+2}(\hat{x}_k)+\dfrac{3(2j+3)}{2j+11}L_{j+4}(\hat{x}_k)-\\ \qquad\dfrac{(2j+3)(2j+5)}{(2j+9)(2j+11)}L_{j+6}(\hat{x}_k), & x\in\Lambda_k,\\ 0, & \text{其他}.\end{cases}$$

则可以验证

$$\left.\begin{aligned}&\mathring{V}_N=\operatorname{span}\{\varphi_0^k,\varphi_1^k,\cdots,\varphi_{N-5}^k\}, \qquad k=1,2,\cdots,K,\\&\mathring{W}_N^k=\operatorname{span}\{\phi_0^k,\phi_1^k,\cdots,\phi_{N-5}^k\}, \qquad k=1,2,\cdots,K.\end{aligned}\right\}\tag{4}$$

令

$$\left.\begin{aligned}&\mathring{V}_{\mathcal{N}}:=\{v;v^k\in\mathring{V}_N^k,k=1,2,\cdots,K\},\\&\mathring{W}_{\mathcal{N}}:=\{v;v^k\in\mathring{W}_N^k,k=1,2,\cdots,K\}.\end{aligned}\right\}\tag{5}$$

当 $k=1,2,\cdots,K-1$ 时,令

$$\Phi_1^k=\begin{cases}\dfrac{3}{5}L_0(\hat{x}_k)+\dfrac{3}{5}L_1(\hat{x}_k)-\dfrac{1}{7}L_2(\hat{x}_k)-\dfrac{1}{10}L_3(\hat{x}_k)+\dfrac{3}{70}L_4(\hat{x}_k), & x\in\Lambda_k,\\ \dfrac{1}{2}L_0(\hat{x}_{k+1})-\dfrac{9}{14}L_1(\hat{x}_{k+1})+\dfrac{1}{6}L_3(\hat{x}_{k+1})-\dfrac{1}{42}L_5(\hat{x}_{k+1}), & x\in\Lambda_{k+1},\\ 0, & \text{其他}.\end{cases}$$

$$\Phi_2^k=\begin{cases}\left(-\frac{3}{10}L_0(\hat{x}_k)-\frac{1}{10}L_1(\hat{x}_k)+\frac{5}{14}L_2(\hat{x}_k)+\frac{1}{10}L_3(\hat{x}_k)-\frac{2}{35}L_4(\hat{x}_k)\right)\frac{h_k}{2}, & x\in\Lambda_k,\\ \left(\frac{1}{5}L_0(\hat{x}_{k+1})-\frac{1}{7}L_1(\hat{x}_{k+1})-\frac{3}{14}L_2(\hat{x}_{k+1})+\frac{1}{6}L_3(\hat{x}_{k+1})+\frac{1}{70}L_4(\hat{x}_{k+1})-\right.\\ \qquad\left.\frac{1}{42}L_5(\hat{x}_{k+1})\right)\frac{h_{k+1}}{2}, & x\in\Lambda_{k+1},\\ 0, & \text{其他}.\end{cases}$$

$$\Phi_3^k=\begin{cases}\left(\frac{1}{15}L_0(\hat{x}_k)-\frac{2}{21}L_2(\hat{x}_k)+\frac{1}{35}L_4(\hat{x}_k)\right)\left(\frac{h_k}{2}\right)^2, & x\in\Lambda_k,\\ 0, & x\in\Lambda_{k+1},\\ 0, & \text{其他}.\end{cases}$$

$$\Psi_1^k=\begin{cases}\frac{1}{2}L_0(\hat{x}_k)+\frac{9}{14}L_1(\hat{x}_k)-\frac{1}{6}L_3(\hat{x}_k)+\frac{1}{42}L_5(\hat{x}_k), & x\in\Lambda_k,\\ \frac{1}{2}L_0(\hat{x}_{k+1})-\frac{9}{14}L_1(\hat{x}_{k+1})+\frac{1}{6}L_3(\hat{x}_{k+1})-\frac{1}{42}L_5(\hat{x}_{k+1}), & x\in\Lambda_{k+1},\\ 0, & \text{其他}.\end{cases}$$

$$\Psi_2^k=\begin{cases}\left(-\frac{1}{5}L_0(\hat{x}_k)-\frac{1}{7}L_1(\hat{x}_k)+\frac{3}{14}L_2(\hat{x}_k)+\frac{1}{6}L_3(\hat{x}_k)-\frac{1}{70}L_4(\hat{x}_k)-\right.\\ \qquad\left.\frac{1}{42}L_5(\hat{x}_k)\right)\frac{h_k}{2}, & x\in\Lambda_k,\\ \left(\frac{1}{5}L_0(\hat{x}_{k+1})-\frac{1}{7}L_1(\hat{x}_{k+1})-\frac{3}{14}L_2(\hat{x}_{k+1})+\frac{1}{6}L_3(\hat{x}_{k+1})+\frac{1}{70}L_4(\hat{x}_{k+1})-\right.\\ \qquad\left.\frac{1}{42}L_5(\hat{x}_{k+1})\right)\frac{h_{k+1}}{2}, & x\in\Lambda_{k+1},\\ 0, & \text{其他}.\end{cases}$$

$$\Psi_3^k=\begin{cases}\left(\frac{1}{30}L_0(\hat{x}_k)+\frac{1}{70}L_1(\hat{x}_k)-\frac{1}{21}L_2(\hat{x}_k)-\frac{1}{45}L_3(\hat{x}_k)+\frac{1}{70}L_4(\hat{x}_k)+\right.\\ \qquad\left.\frac{1}{126}L_5(\hat{x}_k)\right)\left(\frac{h_k}{2}\right)^2, & x\in\Lambda_k,\\ \left(\frac{1}{30}L_0(\hat{x}_{k+1})-\frac{1}{70}L_1(\hat{x}_{k+1})-\frac{1}{21}L_2(\hat{x}_{k+1})+\frac{1}{45}L_3(\hat{x}_{k+1})+\frac{1}{70}L_4(\hat{x}_{k+1})-\right.\\ \qquad\left.\frac{1}{126}L_5(\hat{x}_{k+1})\right)\left(\frac{h_{k+1}}{2}\right)^2, & x\in\Lambda_{k+1},\\ 0, & \text{其他}.\end{cases}$$

通过简单计算可以验证 $k=1,2,\cdots,K-1$ 时，$\Phi_1^k,\Phi_2^k,\Phi_3^k$ 和 $\Psi_1^k,\Psi_2^k,\Psi_3^k$ 满足如下条件，即

$$\Phi_1^k\in V_N,\quad \Phi_1^k(a_k)=1,\quad (\Phi_1^k)'(a_k)=0,\quad (\Phi_1^k)''(a_k)=0,$$
$$\Phi_2^k\in V_N,\quad \Phi_2^k(a_k)=0,\quad (\Phi_2^k)'(a_k)=1,\quad (\Phi_2^k)''(a_k)=0,$$
$$\Phi_3^k\in V_N,\quad \Phi_3^k(a_k)=0,\quad (\Phi_3^k)'(a_k)=0,\quad (\Phi_3^k)''(a_k)=1,\quad \Phi_3^k(\Lambda_{k+1})\equiv 0.$$
$$\Psi_1^k\in W_N,\quad \Psi_1^k(a_k)=1,\quad (\Psi_1^k)'(a_k)=0,\quad (\Psi_1^k)''(a_k)=0,$$
$$\Psi_2^k\in W_N,\quad \Psi_2^k(a_k)=0,\quad (\Psi_2^k)'(a_k)=1,\quad (\Psi_2^k)''(a_k)=0,$$
$$\Psi_3^k\in W_N,\quad \Psi_3^k(a_k)=0,\quad (\Psi_3^k)'(a_k)=0,\quad (\Psi_3^k)''(a_k)=1.$$

而且有

$$V_N=\mathring{V}_N\cup\mathrm{span}\{\Phi_1^k,\Phi_2^k,\Phi_3^k,k=1,2,\cdots,K-1\},$$
$$W_N=\mathring{W}_N\cup\mathrm{span}\{\Psi_1^k,\Psi_2^k,\Psi_3^k,k=1,2,\cdots,K-1\}.$$

将文献[13]求解三阶微分方程的 Petrov-Galerkin 谱元方法的逼近过程推广到式(2)的计算中，详细计算过程包括如下 4 个步骤.

步骤 1　令 $\mathring{\Phi}_1^k(x),\mathring{\Phi}_2^k(x),\mathring{\Phi}_3^k(x)\in\mathring{V}_N$，分别为

$$a(\mathring{\Phi}_j^k,\mathring{v}_N)=-a(\Phi_j^k,\mathring{v}_N),\qquad \forall\,\mathring{v}_N\in\mathring{W}_N,\quad k=1,2,\cdots,K-1,\quad j=1,2,3 \tag{6}$$

的解. 令 $\theta_k=\mathring{\Phi}_1^k+\Phi_1^k$, $\eta_k=\mathring{\Phi}_2^k+\Phi_2^k$, $\sigma_k=\mathring{\Phi}_3^k+\Phi_3^k$,记

$$\mathring{W}_N^{\perp}=\mathrm{span}\{\theta_k,\eta_k,\sigma_k,\ k=1,2,\cdots,K-1\},$$

则 $\mathring{W}_N$ 和 $\mathring{W}_N^{\perp}$ 在 $a(\cdot,\cdot)$意义下是正交的,即

$$a(v_N,\omega_N)=0,\qquad \forall\,v_N\in\mathring{W}_N^{\perp},\quad \forall\,\omega_N\in\mathring{W}_N.$$

步骤 2 找 $\mathring{u}_N\in\mathring{V}_N$,使得

$$a(\mathring{u}_N,\mathring{v}_N)=(f,\mathring{v}_N),\qquad \forall\,\mathring{v}_N\in\mathring{W}_N. \tag{7}$$

步骤 3 求$(u_N(a_k),u'_N(a_k),u''_N(a_k))$, $k=1,2,\cdots,K-1$,使得

$$\sum_{i=1}^{K-1}a(\theta_i,\Psi_j^k)u_N(a_i)+\sum_{i=1}^{K-1}a(\eta_i,\Psi_j^k)u'_N(a_i)+\sum_{i=1}^{K-1}a(\sigma_i,\Psi_j^k)u''_N(a_i)=$$
$$(f,\Psi_j^k)-a(\mathring{u}_N,\Psi_j^k),\qquad k=1,2,\cdots,K-1;\quad j=1,2,3. \tag{8}$$

步骤 4 由式(7),(8)可得

$$a\Big(\mathring{u}_N+\sum_{i=1}^{K-1}(u_N(a_i)\theta_i+u'_N(a_i)\eta_i+u''_N(a_i)\sigma_i),v_N\Big)=(f,v_N),\qquad \forall\,v_N\in W_N. \tag{9}$$

因此, $u_N=\mathring{u}_N+\sum\limits_{i=1}^{K-1}(u_N(a_i)\theta_i+u'_N(a_i)\eta_i+u''_N(a_i)\sigma_i$ 就是方程(2)的解.

问题(2)被分解成两套子区间问题(6),(7)及 Schur 补问题(8). 因此,可以大大提高计算效率.

记 $\mathring{u}_N(x)\,|_{\Lambda_k}=\sum\limits_{i=1}^{K-1}\hat{u}_j^k\varphi_j^k(x)$,并令 $\mathring{v}_N(x)=\phi_j^k(x)$,则式(7)所对应的线性系统可表达为

$$(\alpha A^k-\beta B^k+C^k)\boldsymbol{U}^k=F^k,\qquad k=1,2,\cdots,K.$$

上式中:

$$\boldsymbol{U}^k=(\hat{u}_0^k,\hat{u}_1^k,\cdots,\hat{u}_{N-5}^k)^{\mathrm{T}};$$
$$F^k=(f_0^k,f_1^k,\cdots,f_{N-5}^k),\qquad f_j^k=(f,\phi_j^k)_{\Lambda_k},\qquad j=0,1,\cdots,N-5;$$
$$A_{j,i}^k=(\varphi_i^k,\ \phi_j^k)_{\Lambda_k},\qquad A^k=(A_{j,i}^k)_{0\leqslant i,\ j\leqslant N-5};$$
$$B_{j,i}^k=((\varphi_i^k)'',(\ \phi_j^k)')_{\Lambda_k},\qquad B^k=(B_{j,i}^k)_{0\leqslant i,\ j\leqslant N-5};$$
$$C_{j,i}^k=((\varphi_i^k)'',(\ \phi_j^k)''')_{\Lambda_k},\qquad C^k=(C_{j,i}^k)_{0\leqslant i,\ j\leqslant N-5}.$$

由 Legendre 多项式的正交性可知

$$A_{j,i}^k=\frac{h_k}{2}\begin{cases}-a_{i+5}b_ib_{i+1}, & j=i+5,\\ a_{i+4}b_i, & j=i+4,\\ 2c_{i+1}a_{i+3}+3a_{i+5}b_ib_{i+1}b_{i+4}, & j=i+3,\\ -2a_{i+3}-3a_{i+6}b_i & j=i+2,\\ -a_{i+3}-6a_{i+5}c_{i+1}-3a_{i+6}b_ib_{i+1}c_{i+2}, & j=i+1,\\ a_i+6a_{i+3}b_{i+1}+3a_{i+5}b_ic_{i+1}, & j=i,\\ 3a_{i+3}b_i+6a_{i+4}c_ic_{i+1}+a_{i+5}b_ib_{i+1}c_ic_{i+1}, & j=i-1,\\ -3a_{i+2}-6a_{i+3}(b_i-2a_{i+3})-a_{i+4}b_ic_ic_{i-1}, & j=i-2,\\ -3a_{i+2}+2b_ic_{i-2}-2a_{i+3}c_{i-2}c_{i-1}c_{i+1}, & j=i-3,\\ 3a_{i+1}c_{i-3}+2a_{i+3}c_{i-2}c_{i-3}, & j=i-4,\\ a_{i+3}c_{i-3}c_{i-4}, & j=i-5,\\ -a_ia_{i-5}c_{i-4}, & j=i-6,\\ 0, & \text{其他}.\end{cases}$$

$$B_{j,i}^{k}=(\frac{2}{h_k})^2\begin{cases}2(2i+3)(2i+5), & j=i+2,\\ -2(2i+3)(2i+5), & j=i+1,\\ -4(2i+3)^2 b_{i+1}, & j=i,\\ 4(2i+1)(2i+5)b_i, & j=i-1,\\ 2(2i-1)(2i+1)b_i, & j=i-2,\\ -2(2i-3)(2i-1), & j=i-3,\\ 0, & \text{其他}.\end{cases}$$

$$C_{j,i}^{k}=(\frac{2}{h_k})^4\begin{cases}2(2i+3)^2(2i+5)^2, & j=i,\\ -2(2i+1)(2i+3)^2(2i+5), & j=i-1,\\ 0, & \text{其他}.\end{cases}$$

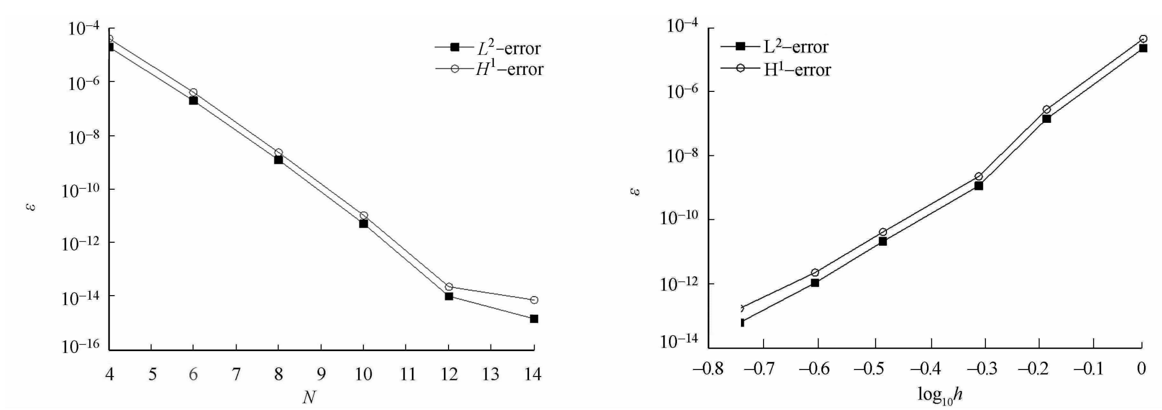
上式中：$a_i=\frac{2}{2i+1}$；$b_i=\frac{2i+3}{2i+7}$；$c_i=\frac{2i+1}{2i+7}$.

2　数值实验

例 1　问题(1)在 $x\in(-1,1)$上的解析解为

$$u(x)=(x-1)^2\sin^2 m\pi x.$$

取计算参数 $\alpha=\beta=m=1$. 在半 log 尺度下，$h=1/2$ 时，L^2-误差 ε 及 H^1-误差随多项式阶数 N 的变化情况，如图 1(a)所示. 由图 1(a)可知：误差随 N 呈指数收敛，这说明对于解析解，数值解具有谱收敛性质. 在 log-log 尺度下，$N=8$ 时，L^2-误差及 H^1-误差随 h 的变化情况，如图 1(b)所示. 由图 1(b)可知：误差关于 h 呈代数衰减.



(a) 误差随 N 的变化情况　　(b) 误差随 h 的变化情况

图 1　误差分别随 N 和 h 的变化图

Fig. 1　Errors as a function of N *and* h respectively

例 2　考虑变系数五阶齐次边值问题，即

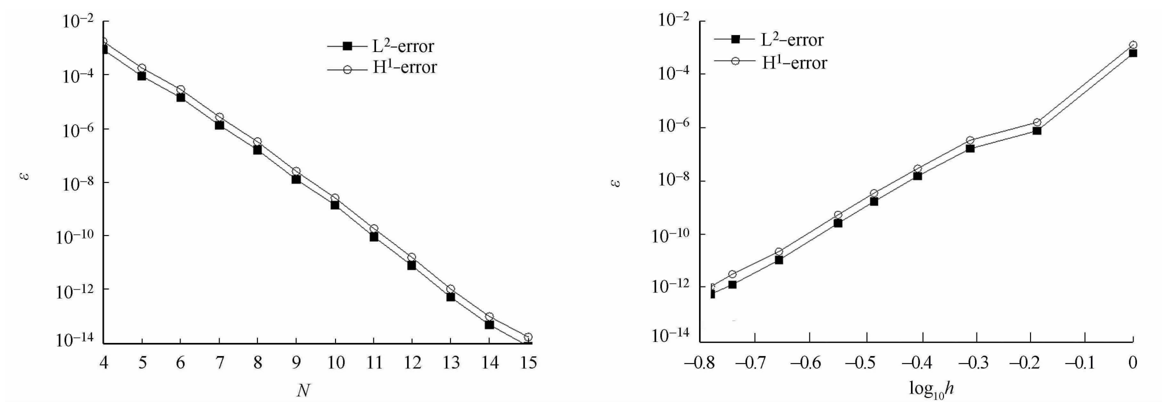
$$\begin{cases}\alpha(x)u+\beta(x)u_x+\gamma u_{x,x,x,x,x}=f, & x\in\Lambda=(-1,1),\\ u(\pm1)=u_x(\pm1)=u_{xx}(1)=0.\end{cases}$$

此时，问题的 Petrov-Galerkin 谱元逼近形式为找 $u_N\in V_N$，使得

$$(\alpha(x)u_N,v_N)+(\beta(x)\partial_x u_N,v_N)-\gamma(\partial_x^2 u_N,\partial_x^3 v_N)=(f,v_N),\qquad \forall v_N\in W_N. \tag{10}$$

该问题同样可采用第 1 节提出的计算过程进行快速计算. 但由于 $\alpha(x)$，$\beta(x)$的存在，弱形式中前两个积分项一般不可精确计算，因此，采用 Gauss-Lobatto 数值积分公式替代.

取 $\alpha(x)=x$，$\beta(x)=\sin 10x$，$\gamma=1$，$u(x)=\sin^3\pi x$ 进行计算. 在半 log 尺度下，$h=1/2$ 时，误差随多项式阶数 N 的变化情况，如图 2(a)所示. 在 log-log 尺度下，$N=8$ 时，误差随 h 的变化情况，如图 2(b)所示. 由图 2 可知：误差随 N 呈指数收敛，误差随 h 呈代数收敛，收敛性质跟常系数方程的计算结果类似. 从而说明 Petrov-Galerkin 谱元逼近法对求解一些变系数五阶常微分方程也是有效的.



(a) 误差随 N 的变化情况 (b) 误差随 h 的变化情况

图 2 依赖于 N 和 h 的误差变化

Fig. 2 Errors as a function corresponding to N and h respectively

3 结束语

利用 Petrov-Galerkin 谱元法对五阶常微分方程进行求解.首先,通过区间剖分,将变分问题转化为一系列子问题;然后,构造恰当的试探函数和检验函数,由此得到稀疏的线性系统再进行求解.数值算例说明 Petrov-Galerkin 谱元逼近法对一些常系数、变系数的五阶常微分方程都是有效的,且具有高精度.

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《华侨大学学报(自然科学版)》简介

BRIEF INTRODUCTION TO JOURNAL OF HUAQIAO UNIVERSITY (NATURAL SCIENCE)

《华侨大学学报(自然科学版)》(下称《学报》)创刊于1980年,是福建省教育厅主管,华侨大学主办,面向国内外公开发行的自然科学综合性学术理论刊物.

《学报》的办刊宗旨是:坚持四项基本原则,贯彻“百花齐放,百家争鸣”和理论与实践相结合的方针,广泛联系海外华侨和港、澳、台、特区的科技信息,及时反映国内尤其华侨大学等高等学府在理论研究、应用研究和开发研究等方面的科技成果,为发展华侨高等教育和繁荣社会主义科技事业服务.

《学报》以创新性、前瞻性、学术性为办刊特色,主要刊登机械工程及自动化、测控技术与仪器、电气工程、电子工程、计算机技术、应用化学、材料与环境工程、化工与生化工程、土木工程、建筑学、应用数学等基础研究和应用研究方面的学术论文,科技成果的学术总结,新技术、新设计、新产品、新工艺、新材料、新理论的论述,以及国内外科技动态的综合评论等内容.

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The Journal has its purpose: adhering to the four cardinal policies, carrying out the principles of the “Flowers Blossom; Schools of Thought Contend” and theory combined with practice, collecting information of science and technology from overseas and those in Hong Kong, Macao, Taiwan and special economic zones and all sides, and in time reflecting the scientific and technological achievements about domestic theoretical research, applied research and development research in our university and others, and serving for development of the overseas Chinese higher education and the socialist prosperity on science and technology.

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