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具有阶段结构和非局部空间效应的竞争系统的稳定性

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摘要：　构造一类具有阶段结构和非局部空间效应影响的两种成年种群个体相互竞争的反应扩散模型. 利用线性稳定化方法和 Redlinger 上下解方法得到该竞争模型的动力性态，并证明模型在边界平衡点和共存平衡点是全局渐近稳定的.

关键词：　阶段结构；非局部空间效应；反应扩散模型；稳定性

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近几十年来，反应扩散方程作为一类重要的半线性抛物型方程，引起了研究者的重视和关注. 本文利用文献[1-3]建模的方法，建立一类具有阶段结构、非线性种内制约关系、非局部空间效应影响、成年种群相互竞争和带有齐次 Neumann 边值的反应扩散模型.

1　反应扩散模型

具体模型为

$$\left.\begin{aligned}
\frac{\partial u_1}{\partial t} &= D_1\frac{\partial^2 u_1}{\partial x^2}+\alpha_1 \mathrm{e}^{-r_1\tau_1}\int_0^\pi G_1(x,y,\tau_1)u_1(t-\tau_1,y)\mathrm{d}y-\beta_1 u_1^{1+\theta_1}(t,x)-c_1u_1(t,x)u_2(t,x),\\
\frac{\partial v_1}{\partial t} &= d_1\frac{\partial^2 v_1}{\partial x^2}+\alpha_1u_1(t,x)-r_1v_1(t,x)-\alpha_1\mathrm{e}^{-r_1\tau_1}\int_0^\pi G_1(x,y,\tau_1)u_1(t-\tau_1,y)\mathrm{d}y,\\
\frac{\partial u_2}{\partial t} &= D_2\frac{\partial^2 u_2}{\partial x^2}+\alpha_2 \mathrm{e}^{-r_2\tau_2}\int_0^\pi G_2(x,y,\tau_2)u_2(t-\tau_2,y)\mathrm{d}y-\beta_2 u_2^{1+\theta_2}(t,x)-c_2u_1(t,x)u_2(t,x),\\
\frac{\partial v_2}{\partial t} &= d_2\frac{\partial^2 v_2}{\partial x^2}+\alpha_2u_2(t,x)-r_2v_2(t,x)-\alpha_2\mathrm{e}^{-r_2\tau_2}\int_0^\pi G_2(x,y,\tau_2)u_2(t-\tau_2,y)\mathrm{d}y,\\
&\frac{\partial u_i}{\partial x}(t,0)=\frac{\partial v_i}{\partial x}(t,0)=\frac{\partial u_i}{\partial x}(t,\pi)=\frac{\partial v_i}{\partial x}(t,\pi)=0,\quad i=1,2,\\
&u_i(t,x)=\varphi_i(t,x),\quad v_i(t,x)=\phi_i(t,x),\quad (t,x)\in[-\tau,0]\times[0,\pi],\quad i=1,2.
\end{aligned}\right\}\tag{1}$$

式(1)中：$t>0, x\in(0,\pi), \tau=\max\{\tau_1,\tau_2\}$，且 $G_i=1/\pi+(2/\pi)\sum_{n=1}^{+\infty}\mathrm{e}^{-d_in^2t}\cos nx\cos ny$ 满足

$$\begin{cases}
\dfrac{\partial G_i}{\partial t}=d_i\dfrac{\partial^2 G_i}{\partial x^2}, & 0<x<\pi,\quad i=1,2,\\
\dfrac{\partial G_i}{\partial t}=0, & x=0,\pi,\\
G_i(x,y,0)=\delta_i(x-y).
\end{cases}$$

其中：$D_i, d_i, \alpha_i, r_i, \tau_i, \beta_i, c_i, \theta_i (i=1,2)$，均大于 0，且生物意义和文献[1,3-4]一样. 由于 $\theta_1,\theta_2>0$，取值的多样性，造成系统动力性态的复杂性，所以只研究当 $\theta_1\geqslant 1, \theta_2\geqslant 1$ 和 $\theta_1<1, \theta_2<1$ 的情况.

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由系统(1)可知,只有成年种群存在竞争,所以只需考虑如下子系统,即

$$\left.\begin{aligned}
&\frac{\partial u_1}{\partial t}=D_1\frac{\partial^2 u_1}{\partial x^2}+\alpha_1 e^{-r_1\tau_1}\int_0^{\pi}G_1(x,y,\tau_1)u_1(t-\tau_1,y)\,dy-\beta_1 u_1^{1+\theta_1}(t,x)-c_1u_1(t,x)u_2(t,x),\\
&\frac{\partial u_2}{\partial t}=D_2\frac{\partial^2 u_2}{\partial x^2}+\alpha_2 e^{-r_2\tau_2}\int_0^{\pi}G_2(x,y,\tau_2)u_2(t-\tau_2,y)\,dy-\beta_2 u_2^{1+\theta_2}(t,x)-c_2u_1(t,x)u_2(t,x),\\
&\qquad\frac{\partial u_i}{\partial x}(t,0)=\frac{\partial u_i}{\partial x}(t,\pi)=0,\quad t>0,\quad i=1,2,\\
&u_i(t,x)=\varphi_i(t,x),\quad (t,x)\in[-\tau,0]\times[0,\pi],\quad i=1,2.
\end{aligned}\right\}\tag{2}$$

2 平衡点和局部稳定性

易知系统(2)有 $E_0=(0,0)$,$E_1=(k_1,0)$,$E_2=(0,k_2)$共3个平衡点. 其中:$k_1=(\frac{\alpha_1 e^{-r_1\tau_1}}{\beta_1})^{1/\theta_1}$;$k_2=(\frac{\alpha_2 e^{-r_2\tau_2}}{\beta_2})^{1/\theta_2}$. 若系统(2)满足条件 $\beta_1k_1^{\theta_1}>c_1k_2$,$\beta_2k_2^{\theta_2}>c_2k_1$,或 $\beta_1k_1^{\theta_1}<c_1k_2$,$\beta_2k_2^{\theta_2}<c_2k_1$,则系统(2)还存在唯一正平衡点 $E=(u_1^*,u_2^*)$. 其中:u_1^*,u_2^* 满足

$$\alpha_1 e^{-r_1\tau_1}-\beta_1(u_1^*)^{\theta_1}-c_1v_1^*=0,\qquad \alpha_2 e^{-r_2\tau_2}-\beta_2(v_1^*)^{\theta_2}-c_2u_1^*=0.\tag{3}$$

对系统(2)做线性化处理,并将$(b_1,b_2)e^{\sigma t+ikx}$代入,可得系统(2)的特征方程为

$$B(\sigma)=\begin{vmatrix} g_1(\sigma,k^2) & c_1u\\ c_2v & g_2(\sigma,k^2)\end{vmatrix},$$

且

$$\left.\begin{aligned}
g_1(\sigma,k^2)&=\sigma+D_1k^2+\beta_1(1+\theta_1)u_1^{\theta_1}+c_1u_2-\alpha_1 e^{-(r_1+\sigma+d_1k^2)\tau_1},\\
g_2(\sigma,k^2)&=\sigma+D_2k^2+\beta_2(1+\theta_2)u_2^{\theta_2}+c_2u_1-\alpha_2 e^{-(r_2+\sigma+d_2k^2)\tau_2}.
\end{aligned}\right\}\tag{4}$$

引理1 系统(2)在 $E_0=(0,0)$处是局部不稳定的.

证明 易知系统(2)在 $E_0=(0,0)$处的特征方程为

$$[\sigma+D_1k^2-\alpha_1 e^{-(r_1+\sigma+d_1k^2)\tau_1}]\times[\sigma+D_2k^2-\alpha_2 e^{-(r_2+\sigma+d_2k^2)\tau_2}]=0.\tag{5}$$

不难看出,当 $k=0$ 时,方程(5)至少存在一个正实部的根,因此引理1得证.

引理2 当 $\beta_1k_1^{\theta_1}>c_1k_2$,$\beta_2k_2^{\theta_2}<c_2k_1$ 时,则系统(1)在 E_1 处局部渐近稳定而在 E_2 处局部不稳定.

证明 易得系统(2)在 $E_1=(k_1,0)$处的特征方程为

$$g_1(\sigma,k^2)g_2(\sigma,k^2)=[\sigma+D_1k^2+(1+\theta_1)\alpha_1 e^{-r_1\tau_1}-\alpha_1 e^{-(r_1+\sigma+d_1k^2)\tau_1}]\times$$
$$[\sigma+D_2k^2+c_2k_1-\alpha_2 e^{-(r_2+\sigma+d_2k^2)\tau_2}]=0,$$

若 $g_1(\sigma,k^2)=0$,有 $\sigma+D_1k^2+(1+\theta_1)\alpha_1 e^{-r_1\tau_1}=\alpha_1 e^{-(r_1+\sigma+d_1k^2)\tau_1}$.

设存在 σ^*,$\mathrm{Re}\,\sigma^*>0$,则有$|\sigma^*+D_1k^2+(1+\theta_1)\alpha_1 e^{-r_1\tau_1}|=|\alpha_1 e^{-(r_1+\sigma+d_1k^2)\tau_1}|\leqslant\alpha_1 e^{-r_1\tau_1}$,与假设矛盾,因此 $\mathrm{Re}\,\sigma<0$. 若 $g_2(\sigma,k^2)$,设存在 σ^*,$\mathrm{Re}\,\sigma^*>0$,则有$|\sigma^*+D_2k^2+c_2k_1|<\alpha_2 e^{-r_2\tau_2}$,与假设矛盾,因此 $\mathrm{Re}\,\sigma<0$,系统(1)在 $E_1=(k_1,0)$处是局部渐近稳定的. 类似可证系统(1)在 $E_2=(0,k_2)$处是局部不稳定,引理2得证. 同理可得如下引理3.

引理3 当 $\beta_1k_1^{\theta_1}<c_1k_2$,$\beta_2k_2^{\theta_2}>c_2k_1$ 时,则系统(2)在 E_2 处局部渐近稳定而在 E_1 处局部不稳定.

引理4 当 $\beta_1k_1^{\theta_1}>c_1k_2$,$\beta_2k_2^{\theta_2}>c_2k_1$ 时,则系统(2)在 $E=(u_1^*,u_2^*)$处是局部渐近稳定的.

证明 由式(3),(4)易得系统(2)在 $E=(u_1^*,u_2^*)$处的特征方程为

$$[\sigma+D_1k^2+\beta_1\theta_1(u_1^*)^{\theta_1}+\alpha_1 e^{-r_1\tau_1}(1-e^{-(\sigma+d_1k^2)\tau_1})]\times$$
$$[\sigma+D_2k^2+\beta_2\theta_2(u_2^*)^{\theta_2}+\alpha_2 e^{-r_2\tau_2}(1-e^{-(\sigma+d_2k^2)\tau_2})]-c_1c_2u_1^*u_2^*=0.$$

令 $\sigma=a+bi$,记 $A_1=a+D_1k^2+\beta_1\theta_1(u_1^*)^{\theta_1}+\alpha_1 e^{-r_1\tau_1}-\alpha_1 e^{-(r_1+d_1k^2+a)\tau_1}\cos b\tau_1$,$A_2=a+D_2k^2+\beta_2\cdot\theta_2(u_2^*)^{\theta_2}+\alpha_2 e^{-r_2\tau_2}-\alpha_2 e^{-(r_2+d_2k^2+a)\tau_2}\cos b\tau_2$,$B_1=b+\alpha_1 e^{-(r_1+d_1k^2+a)\tau_1}\sin b\tau_1$,$B_2=b+\alpha_2 e^{-(r_2+d_2k^2+a)\tau_2}\cdot\sin b\tau_2$. 代入可得$(A_1+B_1i)(A_2+B_2i)=c_1c_2u_1^*u_2^*$. 即 $A_1A_2-B_1B_2=c_1c_2u_1^*u_2^*$,$A_1B_2+A_2B_1=0\Rightarrow A_1A_2\leqslant c_1c_2u_1^*u_2^*$.

假设 $\mathrm{Re}\,\sigma=a\geqslant 0$，可得 $A_1\geqslant D_1k^2+\beta_1\theta_1(u_1^*)^{\theta_1}+\alpha_1\mathrm{e}^{-r_1\tau_1}-\alpha_1\mathrm{e}^{-(r_1+d_1k^2+a)\tau_1}\cos b\tau_1\geqslant\beta_1\theta_1(u_1^*)^{\theta_1}>0$. 同理可得 $A_2\geqslant\beta_2\theta_2(u_2^*)^{\theta_2}>0$，即 $A_1A_2\geqslant\beta_1\beta_2\theta_1\theta_2(u_1^*)^{\theta_1}(u_2^*)^{\theta_2}>0$，从而 $c_1c_2u_1^*u_2^*\geqslant\beta_1\beta_2\theta_1\theta_2(u_1^*)^{\theta_1}\cdot(u_2^*)^{\theta_2}>0\Rightarrow\dfrac{\beta_1\beta_2\theta_1\theta_2(u_1^*)^{\theta_1-1}(u_2^*)^{\theta_2-1}}{c_1c_2}\leqslant 1$ 与条件矛盾[3]. 引理 4 得证.

3　平衡点的全局稳定性

定义 1　系统(2)的一对上下解是一对光滑函数 $\tilde{u}=(\tilde{u}_1,\tilde{u}_2)$，$\hat{u}=(\hat{u}_1,\hat{u}_2)$，且满足

$$\begin{cases}\dfrac{\partial\tilde{u}_i}{\partial t}\geqslant D_i\dfrac{\partial^2\tilde{u}_i}{\partial x^2}+\alpha_i\mathrm{e}^{-r_i\tau_i}\displaystyle\int_0^{\pi}G_i(x,y,\tau_i)\varphi_i(t-\tau_i,y)\mathrm{d}y-\beta_i\tilde{u}_i^{1+\theta_i}(t,x)-c_i\tilde{u}_i(t,x)\tilde{u}_{3-i}(t,x),\\[2ex]\dfrac{\partial\hat{u}_i}{\partial t}\leqslant D_i\dfrac{\partial^2\hat{u}_i}{\partial x^2}+\alpha_i\mathrm{e}^{-r_i\tau_i}\displaystyle\int_0^{\pi}G_i(x,y,\tau_i)\varphi_i(t-\tau_i,y)\mathrm{d}y-\beta_i\hat{u}_i^{1+\theta_i}(t,x)-c_i\hat{u}_i(t,x)\hat{u}_{3-i}(t,x),\\[2ex]\dfrac{\partial\tilde{u}_i}{\partial x}(t,0)\leqslant 0,\dfrac{\partial\tilde{u}_i}{\partial x}(t,\pi)\geqslant 0,\qquad\dfrac{\partial\hat{u}_i}{\partial x}(t,0)\geqslant 0,\dfrac{\partial\hat{u}_i}{\partial x}(t,\pi)\leqslant 0,\qquad t>0,\quad i=1,2,\\[2ex]\hat{u}_i(t,x)=\varphi_i(t,x)\leqslant\tilde{u}_i(t,x),\qquad(t,x)\in[-\tau,0]\times[0,\pi],\quad i=1,2.\end{cases}$$

由文献[5]可得如下引理.

引理 5　如果 $\tilde{u}$ 和 $\hat{u}$ 是系统(2)的一对上下解，且 $\varphi_i(t,x)$ 在 $[-\tau,0]\times[0,\pi]$ 上是 Hölder 连续的，则系统(2)存在唯一的解 $(u_1(t,x),u_2(t,x))$，在 $[-\tau,+\infty]\times[0,\pi]$ 上满足 $\hat{u}_i\leqslant u_i\leqslant\tilde{u}_i(i=1,2)$.

引理 6　如果 $\varphi_i(t,x)$ 在 $[-\tau,0]\times[0,\pi]$ 上是 Hölder 连续的，且 $\varphi_i(t,x)\geqslant 0$，$\varphi_i(0,x)\not\equiv 0(i=1,2)$，则系统(2)存在唯一正解.

证明　取 $K_1\geqslant\max\{\|\varphi_1\|,k_1\}$，$K_2\geqslant\max\{\|\varphi_2\|,k_2\}$，其中 $\|\varphi_i\|=\max\limits_{(t,x)\in[-\tau,0]\times[0,\pi]}|\varphi_i(t,x)|$，$i=1,2$. 显然 $(0,0)$，(K_1,K_2) 为系统(2)的一对上下解. 由引理 5 可知，系统(2)有唯一解 (u_1,u_2) 满足 $0\leqslant u_i\leqslant K_i(i=1,2)$. 下面只需证 $u_i>0(i=1,2)$.

假设存在 $(t^*,x^*)\in[0,+\infty)\times[0,\pi]$ 满足 $u_i(t^*,x^*)=0(i=1,2)$，由系统(2)可得

$$\begin{cases}Lu_i+h_i(t,x)u_i\geqslant 0,\qquad(t,x)\in(0,t^*]\times(0,\pi),\\[1ex]\dfrac{\partial u_i}{\partial t}(0,t)=\dfrac{\partial u_i}{\partial t}(\pi,t)=0,\\[1ex]u_i(x,0)=\varphi_i(x,0),\qquad x\in[0,\pi].\end{cases}$$

其中：$Lu_i=\dfrac{\partial u_i}{\partial t}-D_i\dfrac{\partial^2u_i}{\partial x^2}$；$h_i(t,x)=\beta_iK_i^{\theta_i}+c_iK_iK_{3-i}>0$，$i=1,2$.

此时由文献[6]中的强极大值原理和边界点引理可得，$\varphi_i(0,x)\equiv 0(i=1,2)$ 与引理的条件相矛盾，因此 $\forall(t,x)\in(0,+\infty)\times[0,\pi]$，有 $u_i(t,x)>0(i=1,2)$. 引理得证.

由比较原理，直接可得以下的引理.

引理 7　如果 $u(t,x)$ 是方程

$$\frac{\partial u}{\partial t}=D\frac{\partial^2u}{\partial x^2}+\alpha\mathrm{e}^{-r\tau}\int_0^{\pi}G(x,y,\tau)u(t-\tau,y)\mathrm{d}y-\beta u^{1+\theta_1}(t,x)-cu(t,x),\qquad t>0,\quad x\in[0,\pi]$$

的解，而 $w(t,x)$ 满足

$$\frac{\partial w}{\partial t}\geqslant D\frac{\partial^2w}{\partial x^2}+\alpha\mathrm{e}^{-r\tau}\int_0^{\pi}G(x,y,\tau)w(t-\tau,y)\mathrm{d}y-\beta w^{1+\theta_1}(t,x)-cw(t,x),\qquad t>0,\quad x\in[0,\pi],$$

且当 $t\in[-\tau,0]$ 时，$w(t,x)\geqslant u(t,x)$，则 $\forall t>0$，$w(t,x)\geqslant u(t,x)$.

再由文献[3]引理 3.3 和文献[7]定理 3.1，可得到系统(2)单种群情况的全局吸引性.

引理 8　假设 $\varphi(t,x)$ 在 $[-\tau,0]\times[0,\pi]$ 上是 Hölder 连续的，且 $\varphi(t,x)\geqslant 0$，$\varphi(0,x)\not\equiv 0$，如果 $u(t,x)$ 为方程

$$\begin{cases}\dfrac{\partial u}{\partial t}=D\dfrac{\partial^2u}{\partial x^2}+\alpha\mathrm{e}^{-r\tau}\displaystyle\int_0^{\pi}G(x,y,\tau)u(t-\tau,y)\mathrm{d}y-\\[1ex]\qquad\beta u^{1+\theta_1}(t,x)-Au(t,x),\qquad t>0,\quad x\in[0,\pi],\\[1ex]\dfrac{\partial u}{\partial x}(t,0)=\dfrac{\partial u}{\partial x}(t,\pi)=0,\qquad t>0,\\[1ex]u(t,x)=\varphi(t,x),\qquad(t,x)\in[-\tau,0]\times[0,\pi]\end{cases}$$

的解,且 $\alpha e^{-r\tau}>A\geqslant 0$,则有 $\lim\limits_{t\to+\infty}u(t,x)=(\frac{\alpha e^{-r\tau}-A}{\beta})^{1/\theta_1}$,$x\in[0,\pi]$.

利用文献[1,3,8]中证明全局吸引的方法(交叉迭代法),来证明系统(2)在平衡点 E_1,E_2,E 的全局吸引性.

定理 1 假设 $\beta_1k_1^{\theta_1}>c_1k_2$,$\beta_2k_2^{\theta_2}>c_2k_1$,且 $\varphi_i(t,x)$ 在 $[-\tau,0]\times[0,\pi]$ 上是 Hölder 连续的,$\varphi_i(t,x)\geqslant 0$,$\varphi_i(0,x)\neq 0(i=1,2)$,则系统(2)的解 $u(t,x)=(u_1(t,x),u_2(t,x))$ 满足 $\lim\limits_{t\to+\infty}(u_1(t,x),u_2(t,x))=(u_1^*,u_2^*)$,对 $x\in[0,\pi]$ 是一致成立的.

证明 设 $(u_1(t,x),u_2(t,x))$ 是系统(2)正解,记 $U_1=\lim\limits_{t\to+\infty}\sup\max\limits_{x\in[0,\pi]}u_1(t,x)$,$V_1=\lim\limits_{t\to+\infty}\inf\min\limits_{x\in[0,\pi]}u_1(t,x)$,$U_2=\lim\limits_{t\to+\infty}\sup\max\limits_{x\in[0,\pi]}u_2(t,x)$,$V_2=\lim\limits_{t\to+\infty}\inf\min\limits_{x\in[0,\pi]}u_2(t,x)$. 若要证明 $U_1=V_1=u_1^*$,$U_2=V_2=u_2^*$,则令 $(u_1^{-(1)}(t,x),(u_2^{-(1)}(t,x)))$ 为方程

$$\begin{cases}\dfrac{\partial u_i^{-(1)}}{\partial t}=D_i\dfrac{\partial^2u_i^{-(1)}}{\partial x^2}+\alpha_ie^{-r_i\tau_i}\displaystyle\int_0^\pi G_i(x,y,\tau_i)u_i^{-(1)}(t-\tau_i,y)\mathrm{d}y-\\ \qquad\beta_i(u_i^{-(1)}(t,x))^{1+\theta_i},\qquad t>0,\quad x\in[0,\pi],\\ \dfrac{\partial u_i^{-(1)}}{\partial x}(t,0)=\dfrac{\partial u_i^{-(1)}}{\partial x}(t,\pi)=0,\qquad t>0,\quad i=1,2,\\ u_i^{-(1)}(t,x)=K_i,\qquad (t,x)\in[-\tau,0]\times[0,\pi],\quad i=1,2\end{cases}$$

的解,易知(0,0)和 $(u_1^{-(1)},u_2^{-(1)})$ 为系统(2)的一对上下解.

由引理 5,6 可得,$0<u_1(t,x)\leqslant u_1^{-(1)}(t,x)$,$0<u_2(t,x)\leqslant u_2^{-(1)}(t,x)$. 由引理 8 可得,$\lim\limits_{t\to+\infty}u_i^{-(1)}(t,x)=(\frac{\alpha_ie^{-r_i\tau_i}}{\beta_i})^{1/\theta_i}=:M_1^{u_i}$,$x\in[0,\pi](i=1,2)$,因此,$\forall\varepsilon>0$,$\exists T_{1,1}>0$. 当 $t>T_{1,1}$ 时,$\max\limits_{x\in[0,\pi]}u_i^{-(1)}(t,x)<M_1^{u_i}+\varepsilon(i=1,2)$,从而有 $U_i=\lim\limits_{t\to+\infty}\sup\max\limits_{x\in[0,\pi]}u_1(t,x)\leqslant M_1^{u_i}(i=1,2)$. 令 $(\underline{u}_1^{(1)}(t,x),\underline{u}_2^{(1)}(t,x))$ 为方程

$$\begin{cases}\dfrac{\partial\underline{u}_i^{(1)}}{\partial t}=D_i\dfrac{\partial^2\underline{u}_i^{(1)}}{\partial x^2}+\alpha_ie^{-r_i\tau_i}\displaystyle\int_0^\pi G_i(x,y,\tau_i)\underline{u}_i^{(1)}(t-\tau_i,y)\mathrm{d}y-\\ \qquad\beta_i(\underline{u}_i^{(1)}(t,x))^{1+\theta_i}-c_i\underline{u}_i^{(1)}(t,x)\underline{u}_{3-i}^{(1)}(t,x),\\ \dfrac{\partial\underline{u}_i^{(1)}}{\partial x}(t,0)=\dfrac{\partial\underline{u}_i^{(1)}}{\partial x}(t,\pi)=0,\qquad t>0,\quad i=1,2,\\ \underline{u}_i^{(1)}(t,x)=\dfrac{1}{2}u_i(t,x),\qquad (t,x)\in[-\tau,T_{1,1}]\times[0,\pi],\qquad i=1,2\end{cases}$$

的解,从而 $(u_1^{-(1)},u_2^{-(1)})$ 和 $(\underline{u}_1^{(1)},\underline{u}_2^{(1)})$ 为系统(2)的一对上下解. 由引理 5,6 可得 $\underline{u}_1^{(1)}(t,x)\leqslant u_1(t,x)\leqslant u_1^{-(1)}(t,x)$,$\underline{u}_2^{(1)}(t,x)\leqslant u_2(t,x)\leqslant u_2^{-(1)}(t,x)$.

由条件 $\beta_1k_1^{\theta_1}>c_1k_2$,$\beta_2k_2^{\theta_2}>c_2k_1$ 可得,$\alpha_1e^{-r_1\tau_1}>c_1M_1^{u_2}$,$\alpha_2e^{-r_2\tau_2}>c_2M_1^{u_1}$ 对任意充分小的 $\varepsilon>0$,有 $\alpha_1e^{-r_1\tau_1}>c_1(M_1^{u_2}+\varepsilon)$,$\alpha_2e^{-r_2\tau_2}>c_2(M_1^{u_1}+\varepsilon)$. 考虑方程

$$\begin{cases}\dfrac{\partial w_i^{(1)}}{\partial t}=D_i\dfrac{\partial^2w_i^{(1)}}{\partial x^2}+\alpha_ie^{-r_i\tau_i}\displaystyle\int_0^\pi G_i(x,y,\tau_i)w_i^{(1)}(t-\tau_i,y)\mathrm{d}y-\\ \qquad\beta_i(w_i^{(1)}(t,x))^{1+\theta_i}-c_iw_i^{(1)}(t,x)(M_1^{u_{3-i}}+\varepsilon),\\ \dfrac{\partial w_i^{(1)}}{\partial x}(t,0)=\dfrac{\partial w_i^{(1)}}{\partial x}(t,\pi)=0,\qquad t>T_{1,1},\quad i=1,2,\\ w_i^{(1)}(t,x)=\dfrac{1}{2}u_i(t,x),\qquad (t,x)\in[-\tau,T_{1,1}]\times[0,\pi],\qquad i=1,2,\end{cases}$$

由引理 8 可得,$\lim\limits_{t\to+\infty}w_i^{(1)}(t,x)=(\frac{\alpha_ie^{-r_i\tau_i}-c_i(M_1^{u_{3-i}}+\varepsilon)}{\beta_i})^{1/\theta_i}$,$x\in[0,\pi]$,$i=1,2$,而由引理 7 可得 $\forall\varepsilon>0$,$\exists T_{1,2}\geqslant T_{1,1}$,当 $t>T_{1,1}$ 时,有

$$\min_{x\in[0,\pi]}\underline{u}_i^{(1)}(t,x)>(\frac{\alpha_ie^{-r_i\tau_i}-c_i(M_1^{u_{3-i}}+\varepsilon)}{\beta_i})^{1/\theta_i}-\varepsilon,\qquad i=1,2.$$

由 ε 的任意性可得

$$V_1=\lim_{t\to+\infty}\inf\min_{x\in[0,\pi]}u_i(t,x)\geqslant(\frac{\alpha_ie^{-r_i\tau_i}-c_iM_1^{u_{3-i}}}{\beta_i})^{1/\theta_i}=:N_1^{u_i},\qquad x\in[0,\pi],\quad i=1,2.$$

令$(u_1^{-(2)}(t,x),u_2^{-(2)}(t,x))$为方程

$$\begin{cases}\dfrac{\partial u_i^{-(2)}}{\partial t}=D_i\dfrac{\partial^2 u_i^{-(2)}}{\partial x^2}+\alpha_i\mathrm{e}^{-r_i\tau_i}\displaystyle\int_0^\pi G_i(x,y,\tau_i)u_i^{-(2)}(t-\tau_i,y)\mathrm{d}y-\\ \qquad\beta_i(u_i^{-(2)}(t,x))^{1+\theta_i}-c_iu_i^{-(2)}(t,x)u_{3-i}^{-(2)}(t,x),\\ \dfrac{\partial u_i^{-(2)}}{\partial x}(t,0)=\dfrac{\partial u_i^{-(2)}}{\partial x}(t,\pi)=0,\qquad t>T_{1,2},\quad i=1,2,\\ u_i^{-(2)}(t,x)=K_i,\qquad (t,x)\in[-\tau,T_{1,2}]\times[0,\pi],\qquad i=1,2\end{cases}$$

的解，从而$(\underline{u}_1^{(1)},\underline{u}_2^{(1)})$和$(u_1^{-(2)},u_2^{-(2)})$为系统(2)的一对上下解.

由引理 5,6 可得 $\underline{u}_1^{(1)}(t,x)\leqslant u_1(t,x)\leqslant u_1^{-(2)}(t,x)$，$\underline{u}_2^{(1)}(t,x)\leqslant u_2(t,x)\leqslant u_2^{-(2)}(t,x)$. 考虑如下方程

$$\begin{cases}\dfrac{\partial v_i^{(2)}}{\partial t}=D_i\dfrac{\partial^2 v_i^{(2)}}{\partial x^2}+\alpha_i\mathrm{e}^{-r_i\tau_i}\displaystyle\int_0^\pi G_i(x,y,\tau_i)v_i^{(2)}(t-\tau_i,y)\mathrm{d}y-\beta_i(v_i^{(2)}(t,x))^{1+\theta_i}-c_i(N_1^{u_{3-i}}-\varepsilon),\\ \dfrac{\partial v_i^{(2)}}{\partial x}(t,0)=\dfrac{\partial v_i^{(2)}}{\partial x}(t,\pi)=0,\qquad t>T_{1,2},\quad i=1,2,\\ v_i^{(2)}(t,x)=K_i,\qquad (t,x)\in[-\tau,T_{1,2}]\times[0,\pi],\qquad i=1,2.\end{cases}$$

由引理 8 可得 $\lim\limits_{t\to+\infty}v_i^{(2)}(t,x)=(\dfrac{\alpha_i\mathrm{e}^{-r_i\tau_i}-c_i(N_1^{u_{3-i}}-\varepsilon)}{\beta_i})^{1/\theta_i}$，$x\in[0,\pi]$，$i=1,2$. 由引理 7 可得 $\forall\varepsilon>0$，$\exists T_{2,1}\geqslant T_{1,2}$，当 $t>T_{2,1}$ 时，有

$$\max_{x\in[0,\pi]}u_i^{-(2)}(t,x)\leqslant(\frac{\alpha_i\mathrm{e}^{-r_i\tau_i}-c_i(N_1^{u_{3-i}}-\varepsilon)}{\beta_i})^{1/\theta_i}+\varepsilon,\qquad i=1,2.$$

由 ε 的任意性可得

$$U_i=\limsup_{t\to+\infty}\max_{x\in[0,\pi]}u_i(t,x)\leqslant(\frac{\alpha_i\mathrm{e}^{-r_i\tau_i}-c_iN_1^{u_{3-i}}}{\beta_i})^{1/\theta_i}=:M_2^{u_i},\qquad x\in[0,\pi],\quad i=1,2.$$

令$(\underline{u}_1^{(2)}(t,x),\underline{u}_2^{(2)}(t,x))$为方程

$$\begin{cases}\dfrac{\partial \underline{u}_i^{(2)}}{\partial t}=D_i\dfrac{\partial^2 \underline{u}_i^{(2)}}{\partial x^2}+\alpha_i\mathrm{e}^{-r_i\tau_i}\displaystyle\int_0^\pi G_i(x,y,\tau_i)\underline{u}_i^{(2)}(t-\tau_i,y)\mathrm{d}y-\\ \qquad\beta_i(\underline{u}_i^{(2)}(t,x))^{1+\theta_i}-c_i\underline{u}_i^{(2)}(t,x)\underline{u}_{3-i}^{(2)}(t,x),\\ \dfrac{\partial \underline{u}_i^{(2)}}{\partial x}(t,0)=\dfrac{\partial \underline{u}_i^{(2)}}{\partial x}(t,\pi)=0,\qquad t>T_{2,1},\quad i=1,2,\\ \underline{u}_i^{(2)}(t,x)=\dfrac{1}{2}u_i(t,x),\qquad (t,x)\in[-\tau,T_{2,1}]\times[0,\pi],\qquad i=1,2\end{cases}$$

的解，并考虑方程

$$\begin{cases}\dfrac{\partial w_i^{(2)}}{\partial t}=D_i\dfrac{\partial^2 w_i^{(2)}}{\partial x^2}+\alpha_i\mathrm{e}^{-r_i\tau_i}\displaystyle\int_0^\pi G_i(x,y,\tau_i)w_i^{(2)}(t-\tau_i,y)\mathrm{d}y-\\ \qquad\beta_i(w_i^{(2)}(t,x))^{1+\theta_i}-c_iw_i^{(2)}(t,x)(M_2^{u_{3-i}}+\varepsilon),\\ \dfrac{\partial w_i^{(2)}}{\partial x}(t,0)=\dfrac{\partial w_i^{(2)}}{\partial x}(t,\pi)=0,\qquad t>T_{2,1},\quad i=1,2,\\ w_i^{(2)}(t,x)=\dfrac{1}{2}u_i(t,x),\qquad (t,x)\in[-\tau,T_{2,1}]\times[0,\pi],\qquad i=1,2,\end{cases}$$

由引理 8 可得 $\lim\limits_{t\to+\infty}w_i^{(2)}(t,x)=(\dfrac{\alpha_i\mathrm{e}^{-r_i\tau_i}-c_i(M_2^{u_{3-i}}+\varepsilon)}{\beta_i})^{1/\theta_i}$，$x\in[0,\pi]$，$i=1,2$. 比较定理可得，$\forall\varepsilon>0$，$\exists T_{2,2}>T_{2,1}$，当 $t>T_{2,2}$ 时，有

$$\min_{x\in[0,\pi]}\underline{u}_i^{(2)}(t,x)>(\frac{\alpha_i\mathrm{e}^{-r_i\tau_i}-c_i(M_2^{u_{3-i}}+\varepsilon)}{\beta_i})^{1/\theta_i}-\varepsilon,\qquad i=1,2.$$

由 ε 的任意性可得

$$V_i=\liminf_{t\to+\infty}\min_{x\in[0,\pi]}u_i(t,x)\geqslant(\frac{\alpha_i\mathrm{e}^{-r_i\tau_i}-c_iM_2^{u_{3-i}}}{\beta_i})^{1/\theta_i}=:N_2^{u_i},\qquad x\in[0,\pi],\quad i=1,2,$$

重复以上步骤可得到 4 个数列$\{M_n^{u_1}\}$，$\{M_n^{u_2}\}$，$\{N_n^{u_1}\}$，$\{N_n^{u_2}\}$，$n=1,2,3,\cdots$. 当 $n\geqslant2$ 时，有 $N_n^{u_1}=$

$(\frac{\alpha_1 e^{-r_1\tau_1}-c_1 M_n^{u_2}}{\beta_1})^{1/\theta_1}$, $N_n^{u_2}=(\frac{\alpha_2 e^{-r_2\tau_2}-c_2 M_n^{u_1}}{\beta_2})^{1/\theta_2}$, $M_n^{u_1}=(\frac{\alpha_1 e^{-r_1\tau_1}-c_1 N_{n-1}^{u_2}}{\beta_1})^{1/\theta_1}$, $M_n^{u_2}=(\frac{\alpha_2 e^{-r_1\tau_1}-c_2 N_{n-1}^{u_1}}{\beta_2})^{1/\theta_2}$. 由上面数列的构造过程可知,$\{M_n^{u_1}\}\{M_n^{u_2}\}$是单调递减,$\{N_n^{u_1}\}$,$\{N_n^{u_2}\}$是单调递增的,且满足 $N_n^{u_1}\leqslant V_1\leqslant U_1\leqslant M_n^{u_1}$,$N_n^{u_2}\leqslant V_2\leqslant U_2\leqslant M_n^{u_2}$,故 $\lim\limits_{n\to+\infty} M_n^{u_1}$,$\lim\limits_{n\to+\infty} M_n^{u_2}$,$\lim\limits_{n\to+\infty} N_n^{u_1}$,$\lim\limits_{n\to+\infty} N_n^{u_2}$ 都存在.

再由文献[3]定理 3 可知,$\lim\limits_{t\to+\infty}(u_1(t,x),u_2(t,x))=(u_1^*,u_2^*)$,$x\in[0,\pi]$. 其中:$u_1^*$,$u_2^*$ 满足 $\alpha_1 e^{-r_1\tau_1}-\beta_1(u_1^*)^{\theta_1}-c_1 v_1^*=0$,$\alpha_2 e^{-r_2\tau_2}-\beta_2(v_1^*)^{\theta_2}-c_2 u_1^*=0$. 命题得证.

类似定理1和文献[1]定理3.3,3.4的证明方法,再由文献[3]定理3.1,3.2易得出下面定理.

定理2 假设 $\beta_1 k_1^{\theta_1}>c_1 k_2$,$\beta_2 k_2^{\theta_2}<c_2 k_1$,且 $\varphi_i(t,x)$在$[-\tau,0]\times[0,\pi]$上是 Hölder 连续的,$\varphi_i(t,x)\geqslant 0$,$\varphi_i(0,x)\neq 0(i=1,2)$,则系统(2)的解 $u(t,x)=(u_1(t,x),u_2(t,x))$ 满足 $\lim\limits_{t\to+\infty}(u_1(t,x),u_2(t,x))=(k_1,0)$对 $x\in[0,\pi]$是一致成立的.

定理3 假设 $\beta_1 k_1^{\theta_1}<c_1 k_2$,$\beta_2 k_2^{\theta_2}>c_2 k_1$,且 $\varphi_i(t,x)$在$[-\tau,0]\times[0,\pi]$上是 Hölder 连续的,$\varphi_i(t,x)\geqslant 0$,$\varphi_i(0,x)\neq 0(i=1,2)$,则系统(2)的解 $u(t,x)=(u_1(t,x),u_2(t,x))$ 满足 $\lim\limits_{t\to+\infty}(u_1(t,x),u_2(t,x))=(0,k_2)$对 $x\in[0,\pi]$是一致成立的.

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Stability in Competition Models with Stage Structure and Nonlocal Spatial Effect

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Abstract: In this paper, the author constructs a reaction-diffusion model with stage structure and nonlocal spatial effect, the models with the interaction between the two species and adult members in which are in competition. By using the method of upper-lower solutions due to Redlinger, dynamical behaviors of model are studied. Sharp global stability criteria are established for the coexistence equilibrium as well as the extinction equilibrium.

Keywords: stage structure; nonlocal spatial effect; reaction-diffusion model; stability

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