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定时截尾缺失数据下指数分布的参数AMLE

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摘要 试验数据缺失是产品寿命试验中经常遇到的情况,处理起来比较复杂.当寿命分布为指数分布时,给出寻求定时截尾寿命试验数据缺失场合下,样本分布参数的近似极大似然估计.通过大量的Monte-Carlo数值模拟试验,证实所给方法的可行性.

关键词 近似极大似然估计, 指数分布, 定时截尾, 数据缺失, Taylor展开

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在用统计方法处理实际问题时,常会遇到数据缺失问题,对不完全数据的处理是统计分析的一个重要领域.设产品寿命T服从指数分布,其分布函数$F(t;\theta)=1-e^{-t/\theta}$.其中,$t>0$, $\theta>0$是平均寿命,$\lambda=1/\theta$为失效率.现假定有n个产品同时参加定时截尾试验,试验进行到τ时刻(τ是预先给定的正数)停止.设在τ时刻前有r个产品失效,记相应的失效时间为$t_1\leqslant t_2\leqslant\cdots\leqslant t_r\leqslant\tau$,总试验时间$T=\sum_i t_i+(n-r)\tau$.若由于某种原因造成了数据丢失,不妨设剩下的数据为[1~4] $0<t_{r_1}\leqslant t_{r_2}\leqslant\cdots\leqslant t_{r_k}\leqslant\tau$,即样本为$\{r_1, r_2, \cdots, r_k\}\subset\{1, 2, \cdots, r\}$.本文给出定时截尾缺失数据下,其指数分布参数的近似极大似然估计.

1 参数的近似极大似然估计

用概率元方法[5],求出样本的似然函数$L_n(\theta|t)$.则有

$$L(\theta)=C[1-e^{-t_{r_1}/\theta}]^{r_1-1}\cdot\theta^{-k}e^{-\sum_{i=1}^{k}t_{r_i}/\theta}\prod_{i=1}^{k-1}[e^{-t_{r_i}/\theta}-e^{-t_{r_{i+1}}/\theta}]^{r_{i+1}-r_i-1}\cdot[e^{-t_{r_k}/\theta}-e^{-\tau/\theta}]^{r-r_k}[e^{-\tau/\theta}]^{n-r},$$

而$[e^{-t_{r_k}/\theta}-e^{-\tau/\theta}]^{r-r_k}$可化为$[e^{-t_{r_k}/\theta}-e^{-t_{r_{k+1}}/\theta}]^{r_{k+1}-r_k-1}$,并归并到$\prod_{i=1}^{k-1}[e^{-t_{r_i}/\theta}-e^{-t_{r_{i+1}}/\theta}]^{r_{i+1}-r_i-1}$项中.则上式可化为

$$L(\theta)=C[1-e^{-t_{r_1}/\theta}]^{r_1-1}\cdot\theta^{-k}e^{-\sum_{i}^{k}t_{r_i}/\theta}\prod_{i=1}^{k-1}[e^{-t_{r_i}/\theta}-e^{-t_{r_{i+1}}/\theta}]^{r_{i+1}-r_i-1}\cdot[e^{-\tau/\theta}]^{n-r},\tag{1}$$

$$\ln L(\theta)=\ln c+(r_1-1)\ln(1-e^{-t_{r_1}/\theta})-k\ln\theta-\sum_{i=1}^{k}t_{r_i}/\theta-(n-r)\tau/\theta+\sum_{i=1}^{k}(r_{i+1}-r_i-1)\ln(e^{-t_{r_1}/\theta}-e^{-t_{r_{1+1}}/\theta}).$$

上式可化为

$$\ln L(m)=\ln c+(r_1-1)\ln(1-e^{-mt_{r_1}})+k\ln m-\sum_{i=1}^{k}m\,t_{r_i}-(n-r)m\tau+\sum_{i=1}^{k}(r_{i+1}-r_i-1)\ln(e^{-mt_{r_1}}-e^{-mt_{r_{1+1}}}),$$

$$\frac{\partial(m)}{\partial m}=(r_1-1)\frac{t_{r_1}e^{-mt_{r_1}}}{1-e^{-mt_{r_1}}}+k/m-$$

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$$\sum_{i=1}^{k} t_{r_i} - (n-r)\tau + \sum_{i=1}^{k}(r_{i+1}-r_i-1)\frac{t_{r_{i+1}}e^{-mt_{r_{i+1}}} - t_{r_i}e^{-mt_{r_i}}}{e^{-mt_{r_i}} - e^{-mt_{r_{i+1}}}}.$$

令$\dfrac{\partial(m)}{\partial m}=0$, 则可得

$$(r_1-1)\frac{t_{r_1}e^{-mt_{r_1}}}{1-e^{-mt_{r_1}}} + k/m + \sum_{i=1}^{k}(r_{i+1}-r_i-1)\frac{t_{r_{i+1}}e^{-mt_{r_{i+1}}} - t_{r_i}e^{-mt_{r_i}}}{e^{-mt_{r_i}} - e^{-mt_{r_{i+1}}}} = \sum_{i=1}^{k} t_{r_i} + (n-r)\tau, \tag{2}$$

式(2)的右边$\sum_{i=1}^{k} t_{r_i} + (n-r)\tau$为定值. 令$mt_{r_i}=x_{r_i}$, 等式的左边可化为

$$(r_1-1)\frac{1}{m}\frac{x_{r_1}e^{-x_{r_1}}}{1-e^{-x_{r_1}}} + k/m + \frac{1}{m}\sum_{i=1}^{k}(r_{i+1}-r_i-1)\frac{x_{r_{i+1}}e^{-x_{r_{i+1}}} - x_{r_i}e^{-x_{r_i}}}{e^{-x_{r_i}} - e^{-x_{r_{i+1}}}}. \tag{3}$$

记

$$g(x_{r_1}) = \frac{x_{r_1}e^{-x_{r_1}}}{1-e^{-x_{r_1}}},\qquad h(x_{r_i}, x_{r_{i+1}}) = \frac{x_{r_{i+1}}e^{-x_{r_{i+1}}} - x_{r_i}e^{-x_{r_i}}}{e^{-x_{r_i}} - e^{-x_{r_{i+1}}}},$$

$$p_{r_i} = \frac{r_i}{n+1},\qquad q_{r_i} = 1-p_{r_i},\qquad \zeta_{r_i} = -\ln q_{r_i}.$$

则ζ_i是标准指数分布的p_{r_i}的分位数. 把函数$g(x_{r_1})$与$h(x_{r_i}, x_{r_{i+1}})$分别在点与点ζ_{r_i}与点$(\zeta_{r_{i+1}}, \zeta_i)$处Taylor展开得

$$g(x_{r_1}) \approx d_0 + \beta_0 x_{r_i},$$

$$\alpha_0 = \frac{-q_{r_1}\ln q_{r_1}}{p_{r_1}} + \frac{q_{r_1} + q_{r_1}\ln q_{r_1} - q_{r_1}^2 - 2q_{r_1}^2\ln q_{r_1}^2}{p_{r_1}}\cdot\ln q_{r_1},$$

$$\beta_0 = \frac{q_{r_1} + q_{r_1}\ln q_{r_1} - q_{r_1}^2 - 2q_{r_1}^2\ln q_{r_1}^2}{p_{r_1}}\cdot h(x_{r_i}, x_{r_{i+1}}) \approx \alpha_i + \beta_i x_{r_i} + \gamma_i x_{r_{i+1}},$$

$$\alpha_i = \frac{-q_{r_{i+1}}\ln q_{r_{i+1}} + q_{r_i}\ln q_{r_i}}{q_{r_i} - q_{r_{i+1}}} + \ln q_{r_i}\cdot\frac{-q_{r_i}^2 + (1+\ln q_{r_i} - \ln q_{r_{i+1}})q_{r_i}q_{r_{i+1}}}{(q_{r_i} - q_{r_{i+1}})^2} + \ln q_{r_{i+1}}\cdot\frac{-q_{r_{i+1}}^2 + (1-\ln q_{r_i} + \ln q_{r_{i+1}})q_{r_i}q_{r_{i+1}}}{(q_{r_i} - q_{r_{i+1}})^2},$$

$$\beta_i = \frac{-q_{r_i}^2 + (1+\ln q_{r_i} - \ln q_{r_{i+1}})q_{r_i}q_{r_{i+1}}}{(q_{r_i} - q_{r_{i+1}})^2},$$

$$\gamma_i = \frac{-q_{r_{i+1}}^2 + (1-\ln q_{r_i} + \ln q_{r_{i+1}})q_{r_i}q_{r_{i+1}}}{(q_{r_i} - q_{r_{i+1}})^2}.$$

故式(3)可化为

$$(r_1-1)\frac{1}{m}(\alpha_0 + \beta_0 x_{r_1}) + k/m + \frac{1}{m}\sum_{i=1}^{k}(r_{i+1}-r_i-1)(\alpha_i + \beta_i x_{r_i} + \gamma_i x_{r_{i+1}}).$$

注意到$mt_{r_i}=x_{r_i}$, 对上式进行整理, 可得

$$\frac{1}{m}\Big[(r_1-1)\alpha_0 + k + \sum_{i=1}^{k}(r_{i+1}-r_i-1)\alpha_i\Big] + (r_1-1)\beta_0 t_{r_1} + \sum_{i=1}^{k}(r_{i+1}-r_i-1)(\beta_i t_{r_i} + \gamma_i t_{r_{i+1}})$$

将式(3)代入式(1)中, 可得$\theta=\dfrac{1}{m}$的近似极大似然估计为

$$\theta = \frac{\sum_{i=1}^{k} t_{r_i} + (n-r)\tau - (r_1-1)\beta_0 t_{r_1} - \sum_{i=1}^{k}(r_{i+1}-r_i-1)(\beta_i t_{r_i} + \gamma_i t_{r_{i+1}})}{(r_1-1)\alpha_0 + k + \sum_{i=1}^{k}(r_{i+1}-r_i-1)\alpha_i}. \tag{4}$$

2 Monte-Carlo 模拟

利用式(4)分别对真值θ为1.0, 0.5, 0.25进行了2 000次Monte-Carlo模拟实验, 部分模拟结果如表1所示. 从表中可以看到, 当n固定时, 随k的增大, 精度愈高; 而当k很小时, 即缺失数太大, 参数估计误差偏大. 故我们应尽量避免数据缺失. 总的来说, 在缺失数据数目不太大时, 参数估计的精度还是令人满意的. 利用本文办法可得θ的近似极大似然估计, 部分模拟结果如表1所示.

表 1　Monte-Carlo 部分模拟结果

项目	缺失情况	AMLE	偏差
$\theta=1.0$	$k=15$, 无缺失	0.980	−0.020
$n=20$, $\tau=1.6$, $r=15$	$k=13$, t_1, t_2 缺失; t_1, t_7 缺失	0.950 0.945	−0.050 −0.055
	$k=10$, t_1, t_2, t_6, t_8, t_{10} 缺失	0.905	−0.095
$\theta=0.5$	$k=20$, 无缺失	0.490	−0.010
$n=25$, $\tau=2.5$, $r=20$	$k=17$, t_1, t_2, t_9 缺失; t_3, t_5, t_{12} 缺失	0.511 0.482	0.011 −0.018
	$k=15$, t_2, t_4, t_5, t_8, t_{14} 缺失	0.438	−0.062
$\theta=0.25$	$k=24$, 无缺失	0.248	−0.002
$n=30$, $\tau=5.0$, $r=24$	$k=22$, t_4, t_{20} 缺失; t_3, t_{18} 缺失	0.254 0.245	0.004 −0.005
	$k=20$, t_1, t_5, t_7, t_{17}, t_{22} 缺失	0.239	−0.011

参　考　文　献

1 Balasubranmanian K, Blakrishna N. Estimation for one- and two-parameter exponential distributions under multiple type-Ⅱ censoring[J]. Statistical Papers, 1992, 33: 203～216

2 王乃生, 王玲玲. 定数截尾数据缺失场合下指数分布参数的 Bayes 估计[J]. 应用概率统计, 2001, 8(3): 229～235

3 徐晓岭. 定数截尾缺失数据下 Weibull 分布的统计推断[J]. 应用概率统计, 1997, 12(4): 363～370

4 徐晓岭. 数据缺失下场合三参数 Weibull 分布的参数估计[J]. 上海师范大学学报(自然科学版), 2002, 31(2): 22～30

5 茆诗松, 王静龙, 濮晓龙, 等. 高等数理统计[M]. 北京: 高等教育出版社, 施普林格出版社, 2002. 35～38

Amle for the Parameter of Exponential Distribution under Multiply Type-I Censoring

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Abstract　Compared to type-I censoring, multiply type-I censoring is more general, yet mathematical and numerically complicated when product life time follows exponential distribution. This article provides an approximate maximum likelihood estimation for the parameter of the exponential distribution under multiply type-I censoring. By the Mante-Carlo simulation, the feasibility of this method is proved.

Keywords　exponential distribution, multiply type-I censoring, AMLE, Taylor expansion