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# 恒加应力加速寿命试验的可靠性 非参数统计分析

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**摘要** 提出的可靠性非参数统计分析法, 充分利用恒加应力加速寿命试验的失效个数、失效时间及未失效个数 3 种宝贵的数据信息, 克服传统分析法只利用失效个数和失效时间信息的缺陷, 更合理地对产品的可靠性加以评定. 主要内容: (1)  $\hat{P}_k$  的估计公式; (2)  $P_k < P_{k+1}$  的证明; (3) 恒加应力寿命统计分析; (4) 寿命分布的参数估计; (5) 实例计算与比较.

**关键词** 非参数方法, 失效时间, 数据信息

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## 1 非参数方法

设产品的寿命为  $T$ , 任取  $n$  个样品作寿命试验, 结果是在  $t_i$  处有  $r_i$  个失效, 且  $S_i$  个未失效, 记  $P_i = P_r(T \leq t_i)$ ,  $R_i = 1 - P_i = P_r(T > t_i)$  ( $i = \overline{1, m}$ ),  $t_1 < t_2 < \dots < t_m$ . 则在  $(t_1, t_2, \dots, t_m)$  处试验结果的概率为

$$P_r(r(1) | P_1 P_2 \dots P_m) = \prod_{i=1}^m P_i^{r_i} (1 - P_i)^{S_i}, \quad (1)$$

其中  $S_i = n - \sum_{k=1}^i r_k$ ,  $r(i) = (r_1 r_2 \dots r_m)$ ,  $P_i^{r_i} (1 - P_i)^{S_i}$  表示在  $t_i$  处有  $r_i$  个失效且  $S_i$  个未失效的事件概率 ( $i = \overline{1, m}$ ), 称为核函数. 由贝叶斯假设  $P_i$  为  $r.v.$  且  $P_i \sim U(0, 1)$ , 密度函数  $\pi(P_i) = \begin{cases} 1, & 0 < P_i < 1 \\ 0, & \text{其它} \end{cases}$ ,  $P_i$  ( $i = \overline{1, m}$ ) 互相独立,  $P = (P_1 P_2 \dots P_m)$  为随机向量, 密度函数为

$$\pi(P) = \pi(P_1 P_2 \dots P_m) = \begin{cases} 1, & \text{当 } P \in D, \\ 0, & \text{当 } P \notin D, \end{cases} \quad (2)$$

$$D = \{P: 0 < P_1 < P_2 < \dots < P_m < 1\}.$$

因  $V(D) = m! \int_0^{P_2} dP_1 \int_0^{P_3} dP_2 \dots \int_0^{P_m} dP_m = m! \cdot \frac{1}{m!} = 1$ , 而  $\pi(P) = \begin{cases} \frac{1}{V(D)}, & \text{当 } P \in D \\ 0, & \text{当 } P \notin D \end{cases}$ , 故式(2)成立. 记  $r(1) = (r_1 r_2 \dots r_m)$ , 则  $r(1)$  与  $P$  的联合先验密度为

$$f(P, r(1)) = \begin{cases} k \prod_{i=1}^m P_i^{r_i} (1 - P_i)^{S_i}, & \text{当 } P \in D \\ 0, & \text{当 } P \notin D \end{cases} \quad (k \text{ 是比例常数}). \quad (3)$$

那么,  $P$  的后验密度函数为

$$f(P|r(1)) = \begin{cases} \prod_{i=1}^m P_i^{r_i} (1 - P_i)^{S_i} / \int_D \prod_{i=1}^m P_i^{r_i} (1 - P_i)^{S_i} dP_i, & \text{当 } P \in D, \\ 0, & \text{当 } P \notin D. \end{cases} \quad (4)$$

**定理1**  $P_1$  的经验密度函数为  $f(P_1|r(1)) = W_m^{-1} \sum_{j_m=\beta_m}^{g_m} \sum_{j_{m-1}=\beta_{m-1}}^{g_{m-1}} \sum_{j_2=\beta_2}^{g_2} W(j_m, j_{m-1}, \dots, j_2) \cdot B(P_1|\alpha\beta_1)$ , 其中  $r(1) = (r_1 r_2 \dots r_m)$ , 并有

$$\left. \begin{aligned} \alpha_m &= r_m + 1, \beta_m = S_m + 1, g_m = \alpha_m + \beta_m - 1, S_m = n - \sum_{k=1}^m r_k, \\ \alpha_i &= r_i + g_{i+1} + 1 - j_{i+1}, \beta_i = S_i + 1 + j_{i+1}, g_i = \alpha_i + \beta_i - 1, \\ S_i &= n - \sum_{k=1}^i r_k \quad (i = \overline{1, m}), \quad C_{ji} = \begin{bmatrix} g_i \\ j_i \end{bmatrix} B(\alpha_{i-1}, \beta_{i-1}), \\ g_i &= S_{i-1} + g_{i+1} - 1 \quad (i = \overline{2, m-1}), \quad W(j_m, j_{m-1}, \dots, j_2) = \prod_{i=2}^m C_{ji}. \end{aligned} \right\} \quad (5)$$

**证明**

$$f(P_1|r(1)) = W_m^{*-1} \{P_1^{r_1} (1 - P_1)^{S_1} \int_{P_1}^1 P_2^{r_2} (1 - P_2)^{S_2} dP_2 \dots \int_{P_{m-1}}^1 P_m^{r_m} (1 - P_m)^{S_m} dP_m\},$$

$$W_m^* = \int_D \prod_{i=1}^m P_i^{r_i} (1 - P_i)^{S_i} dP_i, \quad D = \{P: 0 < P_1 < P_2 < \dots < P_m < 1\}.$$

**重复应用恒等式**

$$\begin{aligned} \int_0^1 x^{\alpha-1} (1-x)^{\beta-1} dx &= B(\alpha, \beta) \sum_{j=\beta}^{\alpha+\beta-1} \begin{bmatrix} \alpha+\beta-1 \\ j \end{bmatrix} y^{\alpha-\beta-1-j} (1-y)^j, \\ I_m &= \int_{P_{m-1}}^1 P_m^{r_m} (1 - P_m)^{S_m} dP_m = \int_{P_{m-1}}^1 P_m^{r_m+1-1} (1 - P_m)^{S_m+1-1} dP_m = \\ &= \int_{P_{m-1}}^1 P_m^{\alpha_m-1} (1 - P_m)^{\beta_m-1} dP_m = B(\alpha_m, \beta_m) \sum_{j_m=\beta_m}^{g_m} \begin{bmatrix} g_m \\ j_m \end{bmatrix} P_m^{g_m-j_m} (1 - P_{m-1})^{j_m}, \\ I_{m-1} &= \int_{P_{m-2}}^1 P_m^{r_{m-1}} (1 - P_{m-1})^{S_{m-1}} I_m dP_{m-1} = \\ &= B(\alpha_m, \beta_m) \sum_{j_m=\beta_m}^{g_m} \begin{bmatrix} g_m \\ j_m \end{bmatrix} \int_{P_{m-2}}^1 P_m^{r_{m-1}+g_m+1-j_m-1} (1 - P_{m-1})^{S_{m-1}+1-j_m-1} dP_{m-1} = \\ &= B(\alpha_m, \beta_m) \sum_{j_m=\beta_m}^{g_m} \begin{bmatrix} g_m \\ j_m \end{bmatrix} B(\alpha_{m-1}, \beta_{m-1}) \sum_{j_{m-1}=\beta_{m-1}}^{g_{m-1}} \begin{bmatrix} g_{m-1} \\ j_{m-1} \end{bmatrix} P_m^{g_{m-1}-j_{m-1}} (1 - P_{m-2})^{j_{m-1}}, \\ I_{m-2} &= \int_{P_{m-3}}^1 P_m^{r_{m-2}} (1 - P_{m-2})^{S_{m-2}} I_{m-1} dP_{m-2} = B(\alpha_m, \beta_m) \sum_{j_m=\beta_m}^{g_m} \begin{bmatrix} g_m \\ j_m \end{bmatrix} B(\alpha_{m-1}, \beta_{m-1}) \cdot \\ &\quad \sum_{j_{m-1}=\beta_{m-1}}^{g_{m-1}} \begin{bmatrix} g_{m-1} \\ j_{m-1} \end{bmatrix} P_m^{g_{m-1}-j_{m-1}} (1 - P_{m-2})^{j_{m-1}} \dots \end{aligned} \quad (6)$$

$$\begin{aligned}
I_2 &= \int_{P_1}^1 P_2^2 (1 - P_2)^{S_2} I_3 dP_2 = B(\alpha_m, \beta_m) \sum_{j_m=\beta_m}^{g_m} \binom{g_m}{j_m} B(\alpha_{m-1}, \beta_{m-1}) \cdot \\
&\quad \sum_{j_{m-1}=\beta_{m-1}}^{g_{m-1}} \binom{g_{m-1}}{j_{m-1}} B(\alpha_{m-2}, \beta_{m-2}) \dots B(\alpha_2, \beta_2) \sum_{j_2=\beta_2}^{g_2} \binom{g_2}{j_2} P_1^{g_2-j_2} (1 - P_1)^{j_2}, \\
I_1 &= P_1^r (1 - P_1)^{S_1} I_2 = B(\alpha_n, \beta_m) \sum_{j_m=\beta_m}^{g_m} \binom{g_m}{j_m} B(\alpha_{m-1}, \beta_{m-1}) \cdot \\
&\quad \sum_{j_{m-1}=\beta_{m-1}}^{g_{m-1}} \binom{g_{m-1}}{j_{m-1}} B(\alpha_{m-2}, \beta_{m-2}) \dots B(\alpha_2, \beta_2) \sum_{j_2=\beta_2}^{g_2} \binom{g_2}{j_2} P_1^{r_1+g_2-j_2} (1 - P_1)^{S_1+j_2} = \\
&\quad B(\alpha_n, \beta_m) \sum_{j_m=\beta_m}^{g_m} \binom{g_m}{j_m} B(\alpha_{m-1}, \beta_{m-1}) \sum_{j_{m-1}=\beta_{m-1}}^{g_{m-1}} \binom{g_{m-1}}{j_{m-1}} B(\alpha_{m-2}, \beta_{m-2}) \cdot \\
&\quad \sum_{j_{m-2}=\beta_{m-2}}^{g_{m-2}} \binom{g_{m-2}}{j_{m-2}} B(\alpha_{m-3}, \beta_{m-3}) \dots \sum_{j_2=\beta_2}^{g_2} \binom{g_2}{j_2} B(\alpha_1, \beta_1) B(P_1 | \alpha_1 \beta_1).
\end{aligned}$$

$f(P_1 | r(1)) = I_1 / W_m^*$ . 因  $\int_0^1 f(P_1 | r(1)) dP_1 = 1$  得  $W_m^* = B(\alpha_n, \beta_m) W_m$ , 故得

$$f(P_1 | r(1)) = W_m^{-1} \sum_{j_m=\beta_m}^{g_m} \sum_{j_{m-1}=\beta_{m-1}}^{g_{m-1}} \dots \sum_{j_2=\beta_2}^{g_2} W(j_m j_{m-1} \dots j_2) B(P_1 | \alpha_1 \beta_1).$$

**推论 1** 在二次损失下  $P_1$  的贝叶斯估计为

$$\hat{P}_1 = W_m^{-1} \sum_{j_m=\beta_m}^{g_m} \sum_{j_{m-1}=\beta_{m-1}}^{g_{m-1}} \dots \sum_{j_2=\beta_2}^{g_2} W(j_m j_{m-1} \dots j_2) \left( \frac{\alpha_1}{\alpha_1 + \beta_1} \right). \quad (7)$$

**证明**  $B(P | u, v) = \begin{cases} \frac{1}{B(u, v)} P^{u-1} (1-P)^{v-1}, & 0 < P < 1, \\ 0, & \text{其它} \end{cases}$ , 故  $\hat{P} = E(P | u, v) = \frac{u}{u+v}$ , 立即得

式(7).

**推论 2** 考虑  $0 < P_2 < P_3 < \dots < P_m < 1$ , 记  $r(2) = (r_2 r_3 \dots r_m)$ , 即得

$$f(P_2 | r(2)) = W_m^{-1} \sum_{j_m=\beta_m}^{g_m} \sum_{j_{m-1}=\beta_{m-1}}^{g_{m-1}} \dots \sum_{j_2=\beta_2}^{g_2} W(j_m j_{m-1} \dots j_2) B(P_2 | \alpha_2 \beta_2).$$

在二次损失下,  $P_2$  的贝叶斯估计为

$$\hat{P}_2 = W_m^{-1} \sum_{j_m=\beta_m}^{g_m} \sum_{j_{m-1}=\beta_{m-1}}^{g_{m-1}} \dots \sum_{j_3=\beta_3}^{g_3} W(j_m j_{m-1} \dots j_3) \left( \frac{\alpha_2}{\alpha_2 + \beta_2} \right).$$

一般地, 因  $(r_{i-1}, S_{i-1})$  ( $i = \overline{2, k}$ ) 不能提供  $P_k$  的信息, 故有  $f(P_k | r(k)) = W_m^{-1} \sum_{j_m=\beta_m}^{g_m} \sum_{j_{m-1}=\beta_{m-1}}^{g_{m-1}} \dots$

$\sum_{j_{k+1}=\beta_{k+1}}^{g_{k+1}} W(j_m j_{m-1} \dots j_{k+1}) B(P_k | \alpha_k \beta_k)$ , 其中

$$\begin{aligned}
W_{m-k+1} &= \sum_{j_m=\beta_m}^{g_m} \sum_{j_{m-1}=\beta_{m-1}}^{g_{m-1}} \dots \sum_{j_{k+1}=\beta_{k+1}}^{g_{k+1}} W(j_m j_{m-1} \dots j_{k+1}), \quad r(k) = \\
&\quad (r_k, r_{k+1}, \dots, r_m), \quad (k = \overline{1, m-1}).
\end{aligned}$$

在二次损失下, 有

$$\hat{P}_k = W_{m-k+1}^{-1} \sum_{j_m=\beta_m}^{g_m} \sum_{j_{m-1}=\beta_{m-1}}^{g_{m-1}} \dots \sum_{j_{k+1}=\beta_{k+1}}^{g_{k+1}} W(j_m j_{m-1} \dots j_{k+1}) \left( \frac{\alpha_k}{\alpha_k + \beta_k} \right). \quad (8)$$

因  $f(P_m | r_m) = P_m^{r_m} (1 - P_m)^{S_m} / B(\alpha_m, \beta_m)$ , 故

$$\hat{P}_m = E(P_m | \alpha, \beta_m) = \frac{r_m + 1}{r_m + S_m + 2}. \quad (9)$$

定理 2

$$\hat{P}_k < \hat{P}_{k+1}, (k = \overline{1, m-1}). \quad (10)$$

证明 由式(8)知  $\hat{P}_{k+1} = W_{m-k}^{-1} \sum_{j_m=\beta_m}^{g_m} \sum_{j_{m-1}=\beta_{m-1}}^{g_{m-1}} \cdots \sum_{j_{k+2}=\beta_{k+2}}^{g_{k+2}} W(j_m, j_{m-1}, \dots, j_{k+1}) \cdot$   
 $(\frac{r_{k+1}+g_{k+2}+1-j_{k+2}}{r_{k+1}+g_{k+1}+g_{k+2}+2})$ . 因  $W_{m-k+1}$  中的每个被加项均大于零且  $W_{m-k+1}$  比  $W_{m-k}$  多一重和, 故  
 $W_{m-k} < W_{m-k+1}$ . 有

$$\begin{aligned} \frac{\alpha_k}{\alpha_k + \beta_k} &= \frac{r_k + g_{k+1} + 1 - j_{k+1}}{r_k + g_{k+1} + S_k + 2} \quad \frac{r_k + g_{k+1} + 1 - \beta_{k+1}}{r_k + g_{k+1} + S_k + 2} \\ \frac{r_k + g_{k+1} + 1 - S_{k+1} - 1 - j_{k+2}}{r_k + g_{k+1} + S_k + 2} &= \frac{r_k + g_{k+1} - S_{k+1} - j_{k+2}}{r_k + g_{k+1} + S_k + 2} = \\ &= \frac{r_k + g_{k+2} + S_k + 1 - S_{k+1} - j_{k+2}}{r_k + g_{k+1} + S_k + 2} = \\ \frac{r_k + g_{k+2} + r_{k+1} + 1 - j_{k+2}}{r_k + g_{k+1} + S_k + 2} &= \frac{r_k}{r_k + g_{k+1} + S_k + 2} + \frac{r_{k+1} + g_{k+2} + 1 - j_{k+2}}{r_k + g_{k+1} + S_k + 2}, \\ \sum_{j_{k+1}=\beta_{k+1}}^{g_{k+1}} C_{j_{k+1}} &= \sum_{j_{k+1}=\beta_{k+1}}^{g_{k+1}} \binom{g_{k+1}}{j_{k+1}} B(\alpha_k, \beta_k) = \sum_{j_{k+1}=\beta_{k+1}}^{g_{k+1}} \frac{g_{k+1}!}{j_{k+1}! (g_{k+1} - j_{k+1})!} \cdot \\ &= \frac{(r_{k+1} + g_{k+1} - j_{k+1})! (S_{k+1} + j_{k+1})!}{(r_{k+1} + S_{k+1} + g_{k+1} + 1)!} = \sum_{j_{k+1}=\beta_{k+1}}^{g_{k+1}} \frac{1}{(r_{k+1} + S_{k+1} + g_{k+1} + 1)}. \\ &= \frac{(S_{k+1} + j_{k+1})(S_{k+1} + j_{k+1} - 1) \cdots (j_{k+1} + 1)(r_{k+1} + g_{k+1} - j_{k+1})}{(S_{k+1} + g_{k+1})(S_{k+1} + j_{k+1} - 1) \cdots (g_{k+1} + 1)(r_{k+1} + S_{k+1} - g_{k+1})} \cdot \\ &= \frac{(r_{k+1} + g_{k+1} - j_{k+1} - 1) \cdots (g_{k+1} + 1 - j_{k+1})}{(r_{k+1} + S_{k+1} - g_{k+1} - 1) \cdots (S_{k+1} + g_{k+1} + 1)} < \frac{g_{k+1} - \beta_{k+1}}{r_{k+1} + S_{k+1} + g_{k+1} + 1} = \\ \frac{\alpha_{k+1} - 1}{S_k + g_{k+1} - 1} &= \frac{r_{k+1} - g_{k+2} - j_{k+2}}{S_k + g_{k+1} + 1} < \frac{r_{k+1} + g_{k+2}}{S_k + g_{k+1}} = \frac{r_{k+1} + g_{k+2}}{S_k + S_k + g_{k+2} + 1} \ll 1, \\ \sum_{j_{k+1}=\beta_{k+1}}^{g_{k+1}} C_{j_{k+1}} \left( \frac{\alpha_k}{\alpha_k + \beta_k} \right) &< \frac{r_{k+1} - j_{k+2}}{2S_k + g_{k+2} + 1} \left( \frac{r_k}{r_k + S_k + g_{k+2} + 2} + \right. \\ &\quad \left. \frac{r_{k+1} + g_{k+1} + 1 - j_{k+2}}{r_k + g_{k+1} + S_k + 2} \right) < \frac{r_{k+1} + g_{k+2} + 1 - j_{k+2}}{r_k + 2S_k + g_{k+2} + 3} < \\ &= \frac{r_{k+1} + g_{k+2} + 1 - j_{k+2}}{r_{k+1} + g_{k+2} + S_{k+2} + 2} = \frac{\alpha_{k+1}}{\alpha_{k+1} + \beta_{k+1}}. \end{aligned}$$

最后的第 2 个不等式是因前项的第 1 因子很小, 第 2 因子的第 1 项小于 1, 故得  $\hat{P}_k < \hat{P}_{k+1}$ . 为了便于计算机计算, 可将式(8)改写为如下形式:

$$W_{m-k+1} = \sum_{j_m=\beta_m}^{g_m} \sum_{j_{m-1}=\beta_{m-1}}^{g_{m-1}} \cdots \sum_{j_{k+1}=\beta_{k+1}}^{g_{k+1}} \prod_{i=k+1}^m \frac{g_i!}{g_{i-1}!} \frac{(S_{i-1} + j_i)!}{j_i!} \frac{(r_{i-1} + g_i - j_i)!}{(g_i - j_i)!}, \quad (11)$$

$$I_k = \sum_{j_m=\beta_m}^{g_m} \sum_{j_{m-1}=\beta_{m-1}}^{g_{m-1}} \cdots \sum_{j_{k+1}=\beta_{k+1}}^{g_{k+1}} \prod_{i=k+1}^m \frac{g_i!}{g_{i-1}!} \frac{(S_{i-1} + j_i)!}{j_i!} \cdot$$

$$\frac{(r_{i-1} + g_i - j_i)!}{(g_i - j_i)!} \left( \frac{r_k + g_{k+1} + 1 - j_{k+1}}{r_k + S_k + g_{k+1} + 2} \right). \quad (12)$$

$$\hat{P}_k = I_k / W_{m-k+1}, \quad (k = \overline{1, m-1}), \quad \hat{P}_m = \frac{r_m + 1}{S_m + r_m + 2}.$$

由上述公式可得下面的可靠性指标: (1) 在  $t_k$  处的可靠度为  $\hat{R}_k = 1 - \hat{P}_k, (k = \overline{1, m})$ ; (2) 在  $t_k$  处的失效率为  $\hat{\lambda}_k = \hat{p}_k / t_k, (t_k \text{ 为试验时间})$ <sup>[1]</sup>; (3) 总体的平均失效率估计为  $\hat{\lambda} = \frac{1}{m} \sum_{k=1}^m \hat{\lambda}_k$ .

## 2 恒加应力寿命试验统计分析

设某产品的寿命为  $T$ , 选择比正常应力水平  $\varphi$  较高的  $l$  个应力水平  $\varphi < \varphi < \dots < \varphi$  在  $\varphi$  水平上投放  $n_j$  个样品试验, 其结果如表 1 所示.

表 1 试验数据表

应力水平	样品量	失效时间(失效个数, 未失效个数)	截止时间
$\varphi$	$n_1$	$t_{11}, t_{12}, \dots, t_{1k_1}$	$T_{1M_1}$
		$(r_{11}, S_{11}) (r_{12}, S_{12}) \dots (r_{1k_1}, S_{1k_1})$	$(0, S_{1k_1})$
$\varphi$	$n_2$	$t_{21}, t_{22}, \dots, t_{2k_2}$	$T_{2M_2}$
		$(r_{21}, S_{21}) (r_{22}, S_{22}) \dots (r_{2k_2}, S_{2k_2})$	$(0, S_{2k_1})$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\varphi$	$n_l$	$t_{l1}, t_{l2}, \dots, t_{lk_l}$	$T_{lM_l}$
		$(r_{l1}, S_{l1}) (r_{l2}, S_{l2}) \dots (r_{lk_l}, S_{lk_l})$	$(0, S_{lk_l})$

利用第  $j$  水平上的数据  $(r_{j1}, S_{j1}) (r_{j2}, S_{j2}) \dots (r_{jk_j}, S_{jk_j})$  及  $(0, S_{jk_j}), (j = \overline{1, l})$  逐一地应用式 (11~13) 可以算出  $\hat{P}_{j1}, \hat{P}_{j2}, \dots, \hat{P}_{jM_j}, (j = \overline{1, l})$ . 从而可得  $\hat{R}_{j1}, \hat{R}_{j2}, \dots, \hat{R}_{jk_j}$  及  $\hat{R}_{jM_j}$  与  $t_{j1}, t_{j2}, \dots, t_{jk_j}$  及  $T_{jM_j}$  的失效率为  $\hat{\lambda}_{j1} = \hat{P}_{j1} / t_{j1}, \hat{\lambda}_{j2} = \hat{P}_{j2} / t_{j2}, \dots, \hat{\lambda}_{jk_j} = \hat{P}_{jk_j} / t_{jk_j}$  及  $\hat{\lambda}_{jM_j} = \hat{P}_{jM_j} / T_{jM_j}, (j = \overline{1, l})$ . 于是, 应力水平  $j$  上的平均失效率  $\bar{\lambda}_j = (\sum_{i=1}^{k_j} \hat{\lambda}_{ji} + \hat{\lambda}_{jM_j}) / (k_j + 1), (j = \overline{1, l})$ . 因应力水平越高则对应水平的平均失效率就越大. 故有线性模型  $\ln \bar{\lambda}_j = a + b\varphi_j$  其中  $a, b$  可由数据组  $(\bar{\lambda}_j, \varphi_j) (j = \overline{1, l})$ , 可应用最小二乘法估计出来. 从而, 可以预测正常应力水平  $\varphi$  上的平均失效率  $\bar{\lambda}_0$ .

## 3 寿命分布的参数估计

上节讨论当产品寿命分布未知时, 可以估计产品在正常应力水平上的平均失效率  $\bar{\lambda}_0$ , 但其它的可靠性指标难以获得, 若寿命分布已知, 就可获得更多的可靠性指标.

(1) 若产品寿命  $T \sim f(t, \theta) = \frac{1}{\theta} e^{-t/\theta} (t > 0)$ , 则利用上述方法可算得  $\hat{R}_{j1}, \hat{R}_{j2}, \dots, \hat{R}_{jk_j}, \hat{R}_{jM_j} (j = \overline{1, l})$ . 而  $\hat{R}_{ji} = \hat{R}(t_{ji}) = e^{-t_{ji}/\theta}, \hat{R}_{jM_j} = \hat{R}(T_{jM_j}) = e^{-T_{jM_j}/\theta}, \ln \hat{R}_{ji} = -t_{ji}/\theta (i = \overline{1, k_j}), \ln \hat{R}_{jM_j} = -T_{jM_j}/\theta (i = \overline{1, l})$ , 所以  $\hat{\theta}_j = (\sum_{i=1}^{k_j} t_{ji} + T_{jM_j}) / [- (\sum_{i=1}^{k_j} \ln \hat{R}_{ji} + \ln \hat{R}_{jM_j})] (j = \overline{1, l})$ . 因此, 可得各应力水平上的平均寿命  $\hat{\theta}_1, \hat{\theta}_2, \dots, \hat{\theta}_l$ . 由线性模型  $\ln \theta = a + b\varphi$  及数据组  $(\hat{\theta}_j, \varphi_j) (j = \overline{1, l})$ , 按最小二乘法可求得估计值  $a, b$ , 则  $\ln \theta = a + b\varphi$  对正常应力水平  $\varphi$ , 可求得  $\theta_0 = \exp\{a + b\varphi\}, R_0(t) = e^{-t/\theta_0}, \lambda_0 = 1/\theta_0$  及其它可靠性指标.

(2) 若产品寿命  $T \sim M(t, \alpha, \beta) = \frac{\alpha}{\beta} (\frac{t}{\beta})^{\alpha-1} e^{-(t/\beta)^\alpha} (t > 0)$ , 则  $R(t) = e^{-(t/\beta)^\alpha}$ ,  $\ln(-\ln R(t)) = \alpha \ln t - \alpha \ln \beta$ . 记  $y = \ln(-\ln R(t))$ ,  $a = -\alpha \ln \beta$ ,  $b = \alpha$ ,  $x = \ln t$ , 又得线性模型  $y = a + bx$ . 那么,  $\hat{R}_{ji} = \hat{R}(t_{ji}) (i = \overline{1, k_j})$ ,  $\hat{R}_{jM_j} = \hat{R}(T_{jM_j}) (j = \overline{1, l})$ ,  $x_{ji} = \ln t_{ji} (i = \overline{1, k_j})$ ,  $x_{jM_j} = \ln T_{jM_j} (j = \overline{1, l})$ ,  $y_{ji} = \ln(-\ln R_{ji}) (j = \overline{1, k_j})$ . 按最小二乘法可求得  $\hat{a} = \bar{y}_j - \hat{b}_j \bar{x}_j$ , 其中  $\bar{x}_j = (\sum_{i=1}^{k_j} \ln t_{ji} + \ln T_{jM_j}) / (k_j + 1)$ ,  $\bar{y}_j = \left\{ \sum_{i=1}^{k_j} \ln(-\ln \hat{R}_{ji}) + \ln(-\ln \hat{R}_{jM_j}) \right\} / (k_j + 1)$ ,  $(j = \overline{1, l})$ ,  $\hat{b}_j = \left\{ \sum_{i=1}^{k_j} (x_{ji} - \bar{x}_j)(y_{ji} - \bar{y}_j) + (x_{jM_j} - \bar{x}_j)(y_{jM_j} - \bar{y}_j) \right\} / \left\{ \sum_{i=1}^{k_j} (x_{ji} - \bar{x}_j)^2 (y_{ji} - \bar{y}_j) + (x_{jM_j} - \bar{x}_j)^2 \right\}$ . 再由  $\begin{cases} \hat{a}_j = -\alpha \ln \beta_j \\ \hat{b}_j = \alpha \end{cases}$  解出  $\hat{\alpha}$  与  $\hat{\beta} (j = \overline{1, l})$ , 则得  $\hat{F}_j(t) = 1 - e^{-(t/\hat{\beta})^{\hat{\alpha}}} (j = \overline{1, l})$ . 在加速寿命试验中可应用统计模型. 因此,  $F_n(t) = 1 - \exp\left\{-\left(\frac{t}{\beta}\right)^\alpha\right\}$ , 则变换  $X = T^\alpha \sim F(x) = 1 - \exp(-\frac{x}{\beta^\alpha}) = 1 - \exp(-\frac{x}{\theta})$ ,  $(\theta = \beta^\alpha)$ , 即  $X$  服从指数分布, 因而有阿伦尼斯方程

$$\ln \theta = a_1 + b_1 \varphi \quad (14)$$

当  $\alpha = 1$  时,  $\theta = \beta$ , 则有

$$\ln \beta = a_2 + b_2 \varphi \quad (15)$$

显然, 式(14)与式(15)是两个不同的线性模型. 那么, 应用  $(\hat{\alpha}, \hat{\beta})$  及  $\varphi(j = \overline{1, l})$  可求得  $(\hat{\theta}, \hat{\varphi}) (j = \overline{1, l})$ . 按二乘法, 利用  $(\hat{\beta}, \varphi) (j = \overline{1, l})$ , 可求得  $a_2$  与  $b_2$ ; 利用  $(\hat{\theta}, \varphi) (j = \overline{1, l})$ , 可求得  $a_1$  及  $b_1$ . 这样就得到两个预测方程  $\begin{cases} \ln \beta = \hat{a}_2 + \hat{b}_2 \varphi \\ \ln \theta = \hat{a}_1 + \hat{b}_1 \varphi \end{cases}$  将  $\varphi$  代入即求得  $\hat{\theta}_0$  及  $\hat{\beta}_0$ . 因  $\hat{\theta}_0 = \hat{\beta}_0^{\hat{\alpha}_0}$ , 解得  $\hat{\alpha}_0 = \frac{\ln \hat{\theta}_0}{\ln \hat{\beta}_0}$ , 则有  $F_{W_0}(t) = 1 - \exp\left\{-\left(t/\hat{\beta}_0\right)^{\hat{\alpha}_0}\right\}$ . 由此可求得正常应力水平下的所有可靠性指标. 一般

地, 若  $T \sim F(\frac{t-\alpha}{\beta})$ ,  $F$  的反函数  $F^{-1}$  存在, 则由  $R(t) = 1 - F(\frac{t-\alpha}{\beta})$ ,  $1 - R(t) = F(\frac{t-\alpha}{\beta})$ ,  $\frac{t-\alpha}{\beta} = F^{-1}[1 - R(t)]$ ,  $\frac{t}{\beta} - \frac{\alpha}{\beta} = F^{-1}[1 - R(t)]$ . 记  $y = F^{-1}[1 - R(t)]$ ,  $a = -\frac{\alpha}{\beta}$ ,  $b = \frac{1}{\beta}$ , 同样可得线性模型  $y = a + bt$ . 再用数据组  $(t_i, y_i), (i = \overline{1, m})$ , 按二乘法求得  $\bar{t} = \frac{1}{m} \sum_{i=1}^m t_i$ ,  $\bar{y} = \frac{1}{m} \sum_{i=1}^m F^{-1}(1 - \hat{R}_i)$ .  $\hat{a} = \bar{y} - \hat{b}\bar{t}$ ,  $\hat{b} = S_y/S_x$ , 则从  $\begin{cases} \hat{a} = \alpha/\beta \\ \hat{b} = 1/\beta \end{cases}$  可解得  $\hat{\alpha}$  与  $\hat{\beta}$ .

这类分布还有几何分布、极值 I 型分布、正态分布及对数正态分布等, 均可应用该方法进行分析.

## 4 举例

对某型号的电容器进行温度恒加应力寿命试验, 试验结果列于表 2. 试求在正常温度  $S_0 = 323 \text{ K}$  下, 产品寿命的分布函数及其可靠性特征值. 取  $\mathcal{Q}(S) = \frac{1}{k_0 S}$ ,  $(k_0 = 0.8617 \times 10^{-4})$ ,  $\mathcal{Q}(S_0) = 35.9290$ . 应用公式(8), (9) 算得结果, 如表 3 所示.

注意表 3 应力水平(1)中出现  $P_{15} < P_{14}$ . 这是没有错的. 因表 2 中, 490(1, 16) 和 500(0, 16)

表 2 某电容器温度恒加应力寿命试验

应力水平/K	样品量	失效时间( 失效个数, 未失效个数)	截止时间/ h
358	20	59 79 402 490	500
		( 1, 19) ( 1, 18) ( 1, 17) ( 1, 16)	( 0, 16)
398	16	18 22 47 180	200
		( 1, 15) ( 1, 14) ( 1, 13) ( 1, 12)	( 0, 12)
423	12	11 13 87 90	100
		( 1, 11) ( 1, 10) ( 1, 9) ( 1, 8)	( 0, 8)
448	8	6 9 30 52	60
		( 1, 7) ( 1, 6) ( 1, 5) ( 1, 4)	( 0, 4)

表 3 应力水平(1)~(4)的计算结果

应力水平(1)	$\hat{P}_{1i}$	0.040 74	0.049 91	0.065 97	0.105 26	0.058 82
	$\hat{R}_{1i}$	0.959 26	0.950 09	0.934 03	0.894 74	0.941 18
	$\hat{\lambda}_{1i}$	$6.91 \times 10^{-4}$	$6.32 \times 10^{-4}$	$1.64 \times 10^{-4}$	$2.15 \times 10^{-4}$	$1.18 \times 10^{-4}$
应力水平(2)	$\hat{P}_{2i}$	0.051 09	0.062 97	0.083 64	0.133 33	0.079 23
	$\hat{R}_{2i}$	0.948 91	0.937 03	0.916 36	0.866 67	0.920 77
	$\hat{\lambda}_{2i}$	$2.84 \times 10^{-3}$	$2.86 \times 10^{-3}$	$1.74 \times 10^{-3}$	$7.17 \times 10^{-3}$	$3.96 \times 10^{-3}$
应力水平(3)	$\hat{P}_{3i}$	0.068 49	0.085 29	0.114 21	0.181 78	0.111 11
	$\hat{R}_{3i}$	0.931 51	0.914 71	0.885 79	0.818 22	0.888 89
	$\hat{\lambda}_{3i}$	$6.28 \times 10^{-3}$	$6.56 \times 10^{-5}$	$1.31 \times 10^{-3}$	$1.90 \times 10^{-3}$	$1.11 \times 10^{-3}$
应力水平(4)	$\hat{P}_{4i}$	0.103 52	0.131 56	0.178 91	0.281 80	0.209 00
	$\hat{R}_{4i}$	0.899 65	0.868 44	0.821 09	0.718 20	0.800 00
	$\hat{\lambda}_{4i}$	0.017 25	0.014 62	$5.96 \times 10^{-3}$	$5.42 \times 10^{-3}$	$3.48 \times 10^{-3}$

表示在时间 490 处有 1 个失效, 16 个未失效; 而在 500 处没有失效, 却有 16 个未失效. 其它 3 个水平都有相同的情况. 由表 3 应力水平(1)~(4)的第 3 行数据可以计算出 4 个水平上的平均失效率为  $\hat{\lambda}_1=1.2846 \times 10^{-4}$ ,  $\hat{\lambda}_2=4.1020 \times 10^{-4}$ ,  $\hat{\lambda}_3=1.120 \times 10^{-3}$ ,  $\hat{\lambda}_4=2.7866 \times 10^{-3}$ .

4.1 正常应力水平上平均失效率估计

因为应力水平越高, 则平均失效率就越大, 故有线性模型  $\ln \bar{\lambda}=a+b \varphi(S), \varphi(S)=\frac{1}{k S},(k=0.8617 \times 10^{-4})$ . 应用数据组

$$\ln \bar{\lambda}: -8.9599,-7.7988,-6.7929,-5.8136,$$
$$\varphi: 32.4162,29.1586,27.4353,25.9034,$$

可建立方程  $\ln \bar{\lambda}=6.3675-0.4772 \varphi, r_{\lambda \varphi}=-0.9889$ . 方程可靠. 将  $\varphi\left(S_0\right)=35.929$  代入, 求得  $\lambda_0=2.1 \times 10^{-5}$ . 因寿命分布未知, 没法获得更多的可靠性指标.

4.2 假定寿命分布已知时的可靠性分析

( ) 设  $T \sim f(t, \theta)=\frac{1}{\theta} e^{-t / \theta}, t \geq 0$ . (1) 应用本文方法.  $\hat{R}_{ji}=R\left(t_{ji}\right)=e^{-t_{ji} / \hat{\theta}},(j=\overline{1, l}) ; \hat{\theta}=\left[\sum_{i=1}^{k_i} t_{ji}+T_{j M_j}\right] /\left\{-\left[\sum_{i=1}^{k_i} \ln \hat{R}_{ji}+\ln \hat{R}_{j M_j}\right]\right\}$ . 应用线性模型  $\ln \theta=a+b \varphi$  及数据

$$\ln \theta: 8.43299,7.84485,6.41656,5.05817,$$
$$\varphi: 32.4162,29.1586,27.4353,25.9034,$$

算得  $\ln \hat{\theta} = -7.596\ 42 + 0.505\ 934 \varphi, r_{\varphi} = 0.935\ 0$ . 方程可靠. 将  $\varphi(S_0) = 35.929\ 0$  代入, 求得  $\hat{\theta}_{B_0} = 39\ 384.95$  小时,  $\hat{\lambda}_{B_0} = 2.5 \times 10^{-5}$ . (2) 最大似然估计法.  $\hat{\theta}_j = \left[ \sum_{i=1}^{r_j} t_{ji} + (n_j - r_j) T_{jM_j} \right] / r_j$  ( $j = 1, 4$ );  $\ln \hat{\theta} = 7.722\ 0, 6.504\ 66, 5.528\ 44, 4.433\ 79, \varphi = 32.416\ 0, 29.158\ 6, 27.435\ 3, 25.903\ 4$ , 可建立方程  $\ln \hat{\theta} = -8.171\ 4 + 0.494\ 93 \varphi, r_{\varphi} = 0.987\ 7$ . 方程可靠. 将  $\varphi = 35.929\ 0$  代入得  $\hat{\theta}_{L_0} = 14\ 927.2$ ,  $\hat{\lambda}_{L_0} = 6.7 \times 10^{-5}$ . 显然  $\hat{\theta}_{B_0} > \hat{\theta}_{L_0}$ , 且  $\hat{\theta}_{L_0} = 2.64 \hat{\theta}_{L_0}$ .

( ) 设  $T \sim W(\alpha, \beta, t) = \frac{\alpha}{\beta} (t/\beta)^{\alpha-1} e^{-(t/\beta)^{\alpha}}, t \geq 0$ . (1) 最大似然估计. 有

定理<sup>[1]</sup>  $\alpha, \beta$  的最大似然估计, 由下面方程组解得. 即

$$\ln \beta = \left\{ \ln \left[ \sum_{i=1}^{k_j} t_{ji}^{\alpha} + (n_j - r_j) (T_{jM_j})^{\alpha} \right] - \ln r_j \right\} / \alpha, \quad (16)$$

$$\frac{1}{\alpha} = - \sum_{i=1}^{k_j} \ln t_{ji} / r_j + \left[ \sum_{i=1}^{k_j} t_{ji}^{\alpha} \ln t_{ji} + (n_j - r_j) (T_{jM_j})^{\alpha} \ln T_{jM_j} \right]. \quad (17)$$

应用定理, 对表2中各加速应力水平上的数据代入式(17)进行迭代, 至满意的精度(如  $|\hat{\alpha}_k - \hat{\alpha}_{k+1}| < 0.001$ ). 将  $\hat{\alpha}_k$  代入式(16), 求得  $\hat{\beta}_k$ . 于是, 算得  $\hat{\alpha} = 0.986\ 5, \hat{\alpha}_2 = 0.776\ 4, \hat{\alpha}_3 = 0.870\ 2, \hat{\alpha}_4 = 0.934\ 8; \hat{\beta}_1 = 2\ 305.84, \hat{\beta}_2 = 985.94, \hat{\beta}_3 = 280.03, \hat{\beta}_4 = 87.40$ . 并有  $\ln \beta = 7.743\ 2, 6.893\ 6, 5.6349\ 8, 4.470\ 5; \varphi = 7.743\ 2, 29.158\ 6, 27.435\ 3, 25.903\ 4$ . 建立方程  $\ln \hat{\beta} = -8.032\ 4 + 0.494\ 9 \varphi(S)$ ,  $r_{\varphi} = 0.964\ 4$ . 方程可靠. 将  $\varphi = 35.929\ 0$  代入求得  $\hat{\beta} = 17\ 134.72$ . 因  $\hat{\theta} = \hat{\beta}^{\hat{\alpha}} = 2\ 076.973\ 6, 211.069\ 5, 134.758\ 9, 65.022\ 3; \ln \hat{\theta} = 7.638\ 7, 5.352\ 2, 4.903\ 5, 4.174\ 7; \varphi = 32.416\ 2, 29.158\ 6, 27.435\ 3, 25.903\ 4$ , 建方程  $\ln \hat{\theta} = -9.583\ 1 + 0.525\ 7 \varphi(S)$ ,  $r_{\varphi} = 0.983\ 5$ . 方程可靠. 将  $\varphi(S_0) = 35.929\ 0$  代入, 可得  $\ln \hat{\theta} = 9.304\ 8, \hat{\alpha}_0 = \ln \hat{\theta}_0 / \ln \hat{\beta}_0 = 0.954\ 4$ . 在正常应力  $S_0 = 323\ K$  下, 寿命分布函数

$$F_{L_0}(t) = 1 - \exp\{- (t/17\ 134.72)^{0.954\ 4}\}.$$

由此可求得各种可靠性指标为  $EL(T_0) = 17\ 134.72 \times \Gamma(1 + \frac{1}{0.954\ 4}) = 17\ 516.3$ . (2) 本文的方法. 由变换公式  $y_{ji} = \ln(-\ln R(t_{ji}))$ ,  $x_{ji} = \ln t_{ji}$ , 将表2, 3上的各应力水平的数据变换为表4的数据.

表4 数据变换表

$x_{1i}: 4.077\ 5, 4.368\ 4, 5.996\ 5, 6.194\ 4, 6.214\ 6$	$x_{2i}: 2.890\ 4, 3.091\ 0, 3.850\ 1, 5.225\ 7, 5.298\ 3$
$y_{1i}: -3.179\ 8, -2.972\ 0, -2.684\ 6, -2.196\ 2, -2.803\ 1$	$y_{2i}: -2.948\ 1, -2.732\ 8, -2.437\ 9, -1.944\ 2, -2.494$
$\bar{x}_1 = 5.370\ 5, S_{x_1} = 4.457\ 0, S_{x_1} = 2.111\ 1, \bar{y}_1 = -2.767\ 2$	$\bar{x}_2 = 4.071\ 1, S_{x_2} = 5.242\ 7, S_{x_2} = 2.289\ 7$
$S_{y_1} = 0.546\ 3, S_{y_1} = 0.739\ 1, S_{x_1 y_1} = 1.232\ 2$	$\bar{y}_2 = -2.511\ 5, S_{y_2} = 0.567\ 0, S_{y_2} = 0.753\ 0$
$\hat{b}_1 = 0.276\ 5 = \hat{\alpha}_1, \hat{a}_1 = -4.252\ 1$	$S_{x_2 y_2} = 1.392\ 5, \hat{b}_2 = 0.265\ 6 = \hat{\alpha}_2, \hat{a}_2 = -3.592\ 8$

$x_{3i}: 2.397\ 9, 2.564\ 9, 4.465\ 9, 4.564\ 3; \bar{x}_3 = 3.719\ 7, S_{x_3} = 5.134\ 9, S_{x_3} = 2.266\ 0$

$y_{3i}: -2.645\ 8, -2.417\ 5, -2.109\ 7, -1.606\ 3, -2.138\ 9; \bar{y}_3 = -2.183\ 6, S_{y_3} = 0.609\ 0, S_{y_3} = 0.780\ 4$

$S_{x_3 y_3} = 1.463\ 3, \hat{b}_3 = 0.285\ 0 = \hat{\alpha}_3, \hat{a}_3 = -3.243\ 6$



续表

$x_{4i}$ : 1. 791 8, 2. 197 2, 3. 401 2, 3. 951 2, 4. 094 3,  $\bar{x}_4 = 3. 087 1$ ,  $S_{x_4x_4} = 4. 329 5$ ,  $S_{x_4} = 2. 080 7$

$y_{4i}$ : - 2. 246 7, - 1. 958 6, - 1. 623 9, - 1. 105 6, - 1. 499 9;  $\bar{y}_4 = - 1. 687 0$ ,  $S_{y_4y_4} = 0. 764 0$ ,  $S_{y_4} = 0. 874 1$

$S_{x_4y_4} = 1. 677 9$ ,  $\hat{b}_4 = 0. 387 6 = \hat{\alpha}_4$ ,  $\hat{a}_4 = - 2. 883 4$

从表 4 可得  $\hat{\alpha}_1 = 0. 276 5$ ,  $\hat{\alpha}_2 = 0. 265 6$ ,  $\hat{\alpha}_3 = 0. 285 0$ ,  $\hat{\alpha}_4 = 0. 387 6$ ;  $\hat{a}_1 = - 4. 252 1$ ,  $\hat{a}_2 = - 3. 592 8$ ,  $\hat{a}_3 = - 3. 245 0$ ,  $\hat{a}_4 = - 2. 883 4$ . 由  $\ln \hat{\beta} = - \hat{a}_j / \hat{\alpha}_j = 15. 378 3, 13. 527 1, 11. 386 0, 7. 439 1$ ;  $\mathcal{Q} = 32. 416 2, 29. 158 6, 27. 435 3, 25. 903 4$ ; 算得  $\ln \hat{\beta} = - 21. 394 1 + 1. 160 4 \mathcal{Q}$ ,  $r_{\beta \mathcal{Q}} = 0. 950 9$  方程可靠. 将  $\mathcal{Q} = 35. 929 0$  代入, 求得  $\ln \hat{\beta}_0 = 202 979$ ,  $\hat{\beta}_0 = 653 530 650$ . 下面我们求  $\alpha_0$ , 由  $\hat{\alpha} \ln \hat{\beta} = \ln \hat{\theta} = 4. 252 1, 3. 592 8, 3. 245 0, 2. 883 4$ ;  $\mathcal{Q} = 32. 416 2, 29. 158 6, 27. 435 3, 25. 903 4$ ; 建立方程  $\ln \hat{\theta} = - 2. 497 7 + 0. 208 5 \mathcal{Q}$ ,  $r_{\theta \mathcal{Q}} = 0. 990 5$ . 方程可靠. 将  $\mathcal{Q} = 35. 929 0$  代入方程, 可得  $\ln \hat{\theta}_0 = 4. 993 5$ ,  $\hat{\alpha}_0 = \frac{\ln \hat{\theta}_0}{\ln \hat{\beta}_0} = 0. 246 0$ ,  $E_{\beta_0} T_0 = \hat{\beta}_0 \Gamma(1 + 1/\hat{\alpha}_0) = 653 530 655 \times \Gamma(1 + 1/0. 246 0) = 1. 52 \times 10^{10}$ ,  $E_{\beta_0} T_0 = 867 763 E_L(T_0)$ .

## 5 可靠性指数比较

$$\begin{aligned}\hat{F}_{\beta_0}(t) &= 1 - \exp\left\{-\left[\frac{t}{653\,530\,655}\right]^{0.246\,0}\right\}, \\ \hat{F}_{L_0}(t) &= 1 - \exp\left\{-\left[\frac{t}{17\,134.72}\right]^{0.954\,4}\right\}, \\ \hat{R}_{\beta_0}(t) &= \exp\left\{-\left[\frac{t}{653\,530\,655}\right]^{0.246\,0}\right\}, \\ \hat{F}_{L_0}(t) &= \exp\left\{-\left[\frac{t}{17\,134.72}\right]^{0.954\,4}\right\}.\end{aligned}$$

$\hat{\lambda}_{\beta_0}(t) = 2.94 \times 10^{-3} \cdot t^{-0.754}$ ,  $\hat{\lambda}_{L_0}(t) = 8.7 \times 10^{-5} \cdot t^{-0.046}$ ;  $t_{\beta_0}(0.5) = 653\,530\,655 \times (\ln 2)^{1/0.246\,0} = 14\,729\,871.25$ ,  $t_{L_0}(0.5) = 17\,134 \cdot (\ln 2)^{1/0.954\,4} = 11\,670.2$ . 并有

$t$ :	5 000,	$10^4$ ,	$10^5$ ,	$10^6$ ,
$R_{L_0}(t)$ :	0.734 4,	0.549 8,	$4.6 \times 10^{-3}$ ,	$8.8 \times 10^{-22}$ ,
$R_{\beta_0}(t)$ :	0.946 4,	0.936 7,	0.891 2,	0.816 3,
$t$ :	$10^7$ ,	$10^8$ ,	$10^9$ ,	$10^{10}$ ,
$R_{L_0}(t)$ :	0,	0,	0,	0,
$R_{\beta_0}(t)$ :	0.699 3,	0.532 5,	0.329 5,	0.141 4,
$t$ :	5 000,	$10^4$ ,	$10^5$ ,	$10^6$ ,
$\lambda_{L_0}(t)$ :	$5.9 \times 10^{-5}$ ,	$5.7 \times 10^{-5}$ ,	$5.1 \times 10^{-5}$ ,	$4.6 \times 10^{-5}$ ,
$\lambda_{\beta_0}(t)$ :	$4.8 \times 10^{-6}$ ,	$2.8 \times 10^{-6}$ ,	$4.9 \times 10^{-7}$ ,	$8.8 \times 10^{-8}$ ,
$t$ :	0,	0,	0,	0,
$\lambda_{L_0}(t)$ :	$4.1 \times 10^{-5}$ ,	$3.7 \times 10^{-5}$ ,	$3.4 \times 10^{-5}$ ,	$3.02 \times 10^{-5}$ ,
$\lambda_{\beta_0}(t)$ :	$1.6 \times 10^{-8}$ ,	$2.7 \times 10^{-9}$ ,	$4.8 \times 10^{-10}$ ,	$8.5 \times 10^{-11}$ .

## 6 结束语

从以上的数字特征值比较可见, 本文的估计与最大似然估计<sup>[6]</sup>有非常显著的差异. 多数人会相信似然估计法, 那是因为最大似然估计有很久的历史. 但从整个试验所提供的信息来分析, 可发现似然估计存在的缺陷. 例子中提供的 3 种数据信息为失效时间 16 个点, 未失效时间 4 个点; 失效个数 14 个, 未失效个数累计 224 个. 似然估计只利用失效时间与失效个数的信息, 没有充分利用未失效个数的信息(在公式中只用  $(n_i - r_i) T_{jM_j}$  很少的信息). 因此, 所估计出来的平均寿命等数字特征值偏小. 这显然不符合实际情况. 如最大似然估计的平均寿命  $E_L(T_0) = 17\,516.3$  h, 即 2 a. 显然, 这个估计太低, 无实际意义. 本文的估计其平均寿命为  $1.52 \times 10^{10}$  h (867 763 a). 这个数字显得很大, 但人们常说“电子产品只要不损坏, 旧的与新的一样”, 其含义是指其寿命很长. 以前传统的可靠性评定方法中, 都没有反映出“多长”的问题. 本文恰好验证了这一句话. 有的书说半导体的寿命从理论上说是无限长的, 但由于材料、设计与制造技术等因素的影响无法达到. 如果这一句话是真的, 则本文的结论正好是个例证. 特别指出的是恒加应力加速寿命试验时, 本文的方法仍然适用.

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## Nonparametric Statistical Analysis of the Reliability of Life Test under Constantly Accelerating Stress

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**Abstract** For more reasonably evaluating the reliability of products, the authors put forward the method of nonparametric statistical analysis of reliability. By which the data information of life test under constantly accelerating stress, three valuable data information including failure number and failure time and effective number can be fully utilized; and the imperfection of utilizing only failure number and failure time, as the method of traditional analysis does, can be perfected. The main contents include formula for estimating  $\hat{P}_k$ ; demonstration of  $P_k > P_{k+1}$ ; statistical analysis of life under constantly accelerating stress; parametric estimate of life distribution; and case calculation and comparison.

**Keywords** nonparametric method, failure time, data information.