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对流方程的一族高精度恒稳格式

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摘要　为求解对流方程 $u_t = au_x$ 构造一族新的含3参数3层隐式差分格式(在特殊情况下是2层), 其截断误差至少可达 $O[(\Delta t)^2+(\Delta x)^4]$. 在条件 $\alpha_1=\alpha_3$, $\alpha_2 \le 2\alpha_1$ 或 $\alpha_1\ge 0, \alpha_2\ge 0, \alpha_3\ge 0, \alpha_1>\alpha_3, \alpha_1+\alpha_2+\alpha_3=1, \alpha_2\le 1/2$ 之下, 绝对稳定. 特别地, 当参数 $\alpha_1=\alpha_2, \alpha_3=0$ 时得到一个两层恒稳的差分格式. 所有这些格式都可用追赶法求解, 它包含对流方程的已有文献中的隐式高精度恒稳格式.

关键词　对流方程, 差分格式, 绝对稳定

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1　差分格式的构造

考虑对流方程的初值问题

$$\left.\begin{aligned}&\frac{\partial u}{\partial t} = a\frac{\partial u}{\partial x} \qquad (t>0),\\ &u(x,0)=\varphi(x) \qquad (-\infty<x<+\infty).\end{aligned}\right\} \tag{1}$$

现设问题(1)的解 $u(x,t)$ 充分光滑, 时间步长为 Δt, 空间步长为 Δx. 并设 $r=a\Delta t/\Delta x$, 在网点 (x_m, t_n) 处网格函数 $u(x_m, t_n)$ 记为 u_m^n. 利用 Taylor 展开, 不难验证下列数值微分公式:

$$\left(\frac{\partial u}{\partial t}\right)_m^{n+1} = \frac{1}{2\Delta t}(3u_m^{n+1} - 4u_m^n + u_m^{n-1}) + \frac{(\Delta t)^2}{3}\left(\frac{\partial^3 u}{\partial t^3}\right)_m^{n+1} + \cdots, \tag{2}$$

$$\left(\frac{\partial u}{\partial t}\right)_m^{n} = \frac{1}{2\Delta t}(u_m^{n+1} - u_m^{n-1}) - \frac{(\Delta t)^2}{6}\left(\frac{\partial^3 u}{\partial t^3}\right)_m^{n} + \cdots, \tag{3}$$

$$\left(\frac{\partial u}{\partial t}\right)_m^{n-1} = \frac{1}{2\Delta t}(-u_m^{n+1} + 4u_m^n - 3u_m^{n-1}) + \frac{(\Delta t)^2}{3}\left(\frac{\partial^3 u}{\partial t^3}\right)_m^{n-1} + \cdots, \tag{4}$$

引入记号

$$\Delta_h u_m^n = u_{m+1}^n + 4u_m^n + u_{m-1}^n, \quad L_h u_m^n = u_{m+1}^n - u_{m-1}^n, \tag{5}$$

则有

$$\frac{1}{3}\Delta_h\left(\frac{\partial u}{\partial x}\right)_m^n = \frac{1}{\Delta x}L_h u_m^n + \frac{1}{90}(\Delta x)^4\left(\frac{\partial^5 u}{\partial x^5}\right)_m^n + O(\Delta x)^6. \tag{6}$$

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将式(6)用于方程(1), 便得到下面 3 个差分格式为

$$\frac{1}{3a}\Delta_h\frac{3u_m^{n+1}-4u_m^n+u_m^{n-1}}{2\Delta t}=\frac{1}{\Delta x}L_h u_m^{n+1}, \tag{7}$$

$$\frac{1}{3a}\Delta_h\frac{u_m^{n+1}-u_m^{n-1}}{2\Delta t}=\frac{1}{\Delta x}L_h u_m^{n}, \tag{8}$$

$$\frac{1}{3a}\Delta_h\frac{-u_m^{n+1}+4u_m^n-3u_m^{n-1}}{2\Delta t}=\frac{1}{\Delta x}L_h u_m^{n-1}. \tag{9}$$

它们的局部截断误差, 分别为

$$R_1=\frac{1}{9a}(\Delta t)^2\Delta_h\left(\frac{\partial^3 u}{\partial t^3}\right)_m^{n+1}-\frac{1}{90}(\Delta x)^4\left(\frac{\partial^5 u}{\partial x^5}\right)_m^{n+1}+O(\Delta x)^6,$$

$$R_2=-\frac{1}{18a}(\Delta t)^2\Delta_h\left(\frac{\partial^3 u}{\partial t^3}\right)_m^{n}-\frac{1}{90}(\Delta x)^4\left(\frac{\partial^5 u}{\partial x^5}\right)_m^{n}+O(\Delta x)^6,$$

$$R_3=\frac{1}{9a}(\Delta t)^2\Delta_h\left(\frac{\partial^3 u}{\partial t^3}\right)_m^{n-1}-\frac{1}{90}(\Delta x)^4\left(\frac{\partial^5 u}{\partial x^5}\right)_m^{n-1}+O(\Delta x)^6.$$

由 $\alpha_1\times(7)+\alpha_2\times(8)+\alpha_3\times(9)$, 可得到如下含 3 个参数的 3 层隐式差分格式为

$$\frac{1}{3a}\Delta_h\frac{(3\alpha_1+\alpha_2-\alpha_3)u_m^{n+1}-4(\alpha_1-\alpha_3)u_m^n+(\alpha_1-\alpha_2-3\alpha_3)u_m^{n-1}}{2\Delta t}=$$

$$\frac{1}{\Delta x}L_h(\alpha_1 u_m^{n+1}+\alpha_2 u_m^n+\alpha_3 u_m^{n-1}) \tag{10}$$

便得其相应的局部断误差为

$$R=\alpha_1R_1+\alpha_2R_2+\alpha_3R_3=\frac{2\alpha_1-\alpha_2+2\alpha_3}{3a}(\Delta t)^2\left(\frac{\partial^3 u}{\partial t^3}\right)_m^n-$$

$$\frac{\alpha_1+\alpha_2+\alpha_3}{90}(\Delta x)^4\left(\frac{\partial^5 u}{\partial x^5}\right)_m^n+O[(\Delta t)^3+(\Delta t)^2(\Delta x)^2+(\Delta x)^6].$$

由此可见, 3 层格式(10)的局部截断误差阶至少为 $O[(\Delta t)^2+(\Delta x)^4]$. 若令 $(\Delta t)^2$ 项的系数为 0, 可得局部截断误差阶为 $O[(\Delta t)^3+(\Delta t)^2(\Delta x)^2+(\Delta x)^4]$, 但却不满足后面所得的稳定性条件. 若令 $(\Delta x)^4$ 项的系数为 0, 可得局部截断误差为 $O[(\Delta t)^2+(\Delta x)^6]$. 这时, 由后面所得的稳定性条件之一, 推得稳定性限制为 $\alpha_1=\alpha_3=-\frac{1}{2}\alpha_2$.

2　差分格式的稳定性

为分析差分格式(10)的稳定性. 先叙述如下 Miller 准则[1]. 设 $f(\lambda)$ 为复平面上的 n 次多项式, 即 $f(\lambda)=a_0+a_1\lambda+\cdots+a_n\lambda^n\,(a_0\cdot a_n\neq 0)$. 如果定义多项式

$$f^*(\lambda)=\lambda^n f(1/\bar{\lambda})=\bar{a}_n+\bar{a}_{n-1}\lambda+\cdots+\bar{a}_1\lambda^{n-1}+\bar{a}_0\lambda^n$$

及降幂多项式

$$\tilde{f}(\lambda)=\frac{1}{\lambda}[f^*(0)f(\lambda)-f(0)f^*(\lambda)],$$

其中 $\bar{a}_j$ 为 $a_j\,(j=0,1,\cdots,n)$ 的共轭复数. 因此, Miller 准则为多项式 $f(x)$ 的所有根, 按模小于等于 1 的充要条件有两点. (1) $|f^*(0)|>|f(0)|$, 且 $\tilde{f}(x)=0$ 只有按模小于等于 1 的根. (2) $\tilde{f}\equiv 0$ 且 $f'(\lambda)=0$ 的所有根按模小于等于 1. 用 Fourier 方法[2], 分析格式(10)的稳定性. 令 u_m

$=\lambda^n e^{im\theta}(i=\sqrt{-1},\ \theta<\pi)$, 有

$$F \xlongequal{令} \frac{1}{6}e^{-im\theta}\Delta_h e^{im\theta} = \frac{1}{3}(\cos\theta + 2) > 0 \tag{11}$$

$$H \xlongequal{令} re^{-im\theta}L_h e^{im\theta} = (2r\sin\theta)\,i \xlongequal{令} gi, \qquad g = 2r\sin\theta. \tag{12}$$

按文[3]中理论, 式(10)的传播矩阵为

$$\boldsymbol{G}(m,\Delta t) = \begin{pmatrix} -B/A & -C/A \\ 1 & 0 \end{pmatrix}.$$

其特征方程为 $A\lambda^2 + B\lambda + C = 0$, 其中

$$A = (3\alpha_1 + \alpha_2 - \alpha_3)F - \alpha_1 H = (3\alpha_1 + \alpha_2 - \alpha_3)F - \alpha_1 g i,$$

$$B = -4(\alpha_1 - \alpha_3)F - \alpha_2 H = -4(\alpha_1 - \alpha_3)F - \alpha_2 g i,$$

$$C = (\alpha_1 - \alpha_2 - 3\alpha_3)F - \alpha_3 H = (\alpha_1 - \alpha_2 - 3\alpha_3)F - \alpha_3 g i.$$

$$f(\lambda) = [(3\alpha_1 + \alpha_2 - \alpha_3)F - \alpha_1 g i]\lambda^2 + [-4(\alpha_1 - \alpha_3)F - \alpha_2 g i]\lambda + (\alpha_1 - \alpha_2 - 3\alpha_3)F - \alpha_3 g i, \tag{13}$$

$$f(0) = (\alpha_1 - \alpha_2 - 3\alpha_3)F - \alpha_3 g i, \tag{14}$$

$$f^*(\lambda) = [(\alpha_1 - \alpha_2 - 3\alpha_3)F + \alpha_3 g i]\lambda^2 + [-4(\alpha_1 - \alpha_3)F + \alpha_2 g i]\lambda + (3\alpha_1 + \alpha_2 - \alpha_3)F + \alpha_1 g i, \tag{15}$$

$$f^*(0) = (3\alpha_1 + \alpha_2 - \alpha_3)F + \alpha_1 g i. \tag{16}$$

由式(13)～(16), 可得降幂多项式为

$$\tilde{f}(\lambda) = \frac{f^*(0)f(\lambda) - f(0)f^*(\lambda)}{\lambda} = [8(\alpha_1 + \alpha_2 + \alpha_3)(\alpha_1 - \alpha_3)F^2 + (\alpha_1 - \alpha_3)(\alpha_1 + \alpha_3)g^2]\lambda + (\alpha_1 - \alpha_3)[-8(\alpha_1 + \alpha_2 + \alpha_3)F^2 + \alpha_2 g^2 - 4(\alpha_1 + \alpha_2 + \alpha_3)Fgi]. \tag{17}$$

下面分两种情况进行讨论.

(i) 当 $\alpha_1 = \alpha_3$, $\tilde{f}(\lambda) \equiv 0$, 此时

$$f(\lambda) = [(2\alpha_1 + \alpha_2)F - \alpha_1 g i]\lambda^2 - (\alpha_2 g i)\lambda + [(-\alpha_2 - 2\alpha_1)F - \alpha_1 g i],$$

$$\tilde{f}'(\lambda) = 2[(2\alpha_1 + \alpha_2)F - \alpha_1 g i]\lambda - \alpha_2 g i \xlongequal{令} 0.$$

注意到式(11),(12), 当 $\alpha_2 \le 2\alpha_1$ 时, $\tilde{f}'(\lambda)$ 的根模满足

$$\lambda^2 = \frac{\alpha_2^2 g^2}{4(2\alpha_1 + \alpha_2)^2 F^2 + 4\alpha_1^2 g^2} < 1,$$

故 $f'(\lambda)$ 的根模 $\lambda < 1$. 由 Miller 准则知, $f(\lambda)$ 的根模 $\lambda < 1$. 从而, 当 $\alpha_1 = \alpha_3$, $\alpha_2 \le 2\alpha_1$ 时, 差分格式(10)绝对稳定.

(ii) 由式(14)及式(16), 可知

$$f^*(0)^2 = (3\alpha_1 + \alpha_2 - \alpha_3)^2 F^2 + \alpha_1^2 g^2, \quad f(0)^2 = (\alpha_1 - \alpha_2 - 3\alpha_3)^2 F^2 + \alpha_3^2 g^2,$$

$$f^*(0)^2 - f(0)^2 = 8(\alpha_1 + \alpha_2 + \alpha_3)(\alpha_1 - \alpha_3)F^2 + (\alpha_1 - \alpha_3)(\alpha_1 + \alpha_3)g^2.$$

当 $\alpha_1 \ge 0, \alpha_2 \ge 0, \alpha_3 \ge 0$ 且 $\alpha_1 > \alpha_3$ 时, 有 $f^*(0) > f(0)$. 由式(17)可知, 当 $\alpha_1 + \alpha_2 + \alpha_3 = 1$, $\alpha_2 \le 1/2$ 时, $\tilde{f}(\lambda) = 0$ 的根为

$$\lambda^2 = \frac{(8F^2 - \alpha_2 g^2)^2 + 16F^2 g^2}{(8F^2 - \alpha_2 g^2)^2 + 16F^2 g^2 + (1 - 2\alpha_2)g^4} \le 1$$

当 $\alpha_2 < \frac{1}{2}$时, 上述不等式严格成立. 当 $\alpha_2 = \frac{1}{2}$时, 上述不等式成为等式. 但此时, $f(\lambda)$只有单根 $\lambda=1$. 由 Miller 准则可知, 当 $\alpha_1 \geq 0, \alpha_2 \geq 0, \alpha_3 \geq 0, \alpha_1 > \alpha_3, \alpha_2 \leq \frac{1}{2}, \alpha_1+\alpha_2+\alpha_3=1$ 时 $\tilde{f}(\lambda)$ 的根模 $\lambda \leq 1$ 且 $\lambda=1$ 时只有单根. 因此, 差分格式(10) 也稳定.

综上所述, 便得如下基本定理:

定理　当参数 $\alpha_1=\alpha_3$, $\alpha_2 \leq 2\alpha_1$ 或 $\alpha_1 \geq 0, \alpha_2 \geq 0, \alpha_3 \geq 0, \alpha_1 > \alpha_3, \alpha_1+\alpha_2+\alpha_3=1, \alpha_2 \leq \frac{1}{2}$时, 关于对流方程初值问题(1), 差分格式(10) 绝对稳定.

本定理的结论, 对于对流方程初边值问题也适用, 但边界条件应按文[3]给出的才正确. 即当 $a<0$ 时, 应给出左条件 $u(0,t)=f(x)$. 当 $a>0$ 时, 仅应给出右边条件 $u(L,t)=f(x)$.

3　若干特例

对参数偶$(\alpha_1, \alpha_2, \alpha_3)$的不同选取, 便可得到不同的差分格式, 下面略举差分格式(10) 的几个特例.

特例 **1**　记 $\alpha_1-\alpha_2=\alpha, \alpha_2=\frac{1}{2}-2\beta$, 则由 $\alpha_1+\alpha_2+\alpha_3=1$. 推出: (Ⅰ) $3\alpha_1+\alpha_2-\alpha_3=2\alpha+1$; (Ⅱ) $\alpha-3\alpha_3-\alpha_2=2\alpha-1$; (Ⅲ) $\alpha_1=\frac{1}{4}+\frac{\alpha}{2}+\beta$; (Ⅳ) $\alpha_3=\frac{1}{4}-\frac{\alpha}{2}+\beta$. 此时, 格式(10) 成为 3 层 9 点隐式格式. 有

$$\frac{1}{6\Delta t}\Delta_h[(2\alpha+1)u_m^{n+1}-4\alpha u_m^n+(2\alpha-1)u_m^{n-1}]=$$

$$\frac{a}{\Delta x}L_h[(\frac{1}{4}+\frac{\alpha}{2}+\beta)u_m^{n+1}-(\frac{1}{2}-\beta)u_m^n+(\frac{1}{4}-\frac{\alpha}{2}-\beta)u_m^{n+1}].$$

这就是文[4]的差分格式.

$$\frac{1}{6\Delta t}[(\alpha+\frac{1}{2})u_{m+1}^{n+1}-2\alpha u_{m+1}^n+(\alpha-\frac{1}{2})u_{m+1}^{n-1}]+$$

$$\frac{4}{6\Delta t}[(\alpha+\frac{1}{2})u_m^{n+1}-2\alpha u_m^n+(\alpha-\frac{1}{2})u_m^{n-1}]+$$

$$\frac{1}{6\Delta t}[(\alpha+\frac{1}{2})u_{m-1}^{n+1}-2\alpha u_{m-1}^n+(\alpha-\frac{1}{2})u_{m-1}^{n-1}]=$$

$$(\frac{1}{4}+\frac{1}{2}\alpha+\beta)a\frac{u_{m+1}^{n+1}-u_{m-1}^{n+1}}{2\Delta x}+(\frac{1}{2}-2\beta)a\frac{u_{m+1}^n-u_{m-1}^n}{2\Delta x}+$$

$$(\frac{1}{4}-\frac{1}{2}\alpha+\beta)a\frac{u_{m+1}^{n-1}-u_{m-1}^{n-1}}{2\Delta x}, \tag{18}$$

它对任何非负参数 $\alpha \geq 0$ 和 $\beta \geq 0$ 均稳定.

特例 **2**　当 $\alpha_1=\alpha_2, \alpha_3=0$ 时为 2 层 6 点恒稳格式, 即文[4]的格式(4). 有

$$\frac{1}{6\Delta t}(u_{m+1}^{n+1}-u_{m+1}^n)+\frac{4}{6\Delta t}(u_m^{n+1}-u_m^n)+\frac{1}{6\Delta t}(u_{m-1}^{n+1}-u_{m-1}^n)=$$

$$\frac{a}{2}\frac{u_{m+1}^{n+1}-u_{m-1}^{n+1}}{2\Delta x}+\frac{a}{2}\frac{u_{m+1}^n-u_{m-1}^n}{2\Delta x}. \tag{19}$$

特例 **3**　当 $\alpha_1=\alpha_3=\frac{1}{2}, \alpha_2=\frac{3}{4}$时为 3 层 8 点恒稳格式. 有

$$\frac{7}{24\Delta t}(u_{m+1}^{n+1}-u_{m+1}^{n-1})+\frac{7}{6\Delta t}(u_{m}^{n+1}-u_{m}^{n-1})+\frac{7}{24\Delta t}(u_{m-1}^{n+1}-u_{m-1}^{n-1})=$$
$$a\frac{u_{m+1}^{n+1}-u_{m-1}^{n+1}}{2\Delta x}+\frac{3}{2}a\frac{u_{m+1}^{n}-u_{m-1}^{n}}{2\Delta x}+a\frac{u_{m+1}^{n-1}-u_{m-1}^{n-1}}{2\Delta x}. \tag{20}$$

特例 4 当 $\alpha_1=\alpha_3=\frac{1}{2},\alpha_2=\frac{1}{5}$时为 3 层 8 点恒稳格式．有

$$\frac{2}{15\Delta t}(u_{m+1}^{n+1}-u_{m+1}^{n-1})+\frac{8}{15\Delta t}(u_{m}^{n+1}-u_{m}^{n-1})+\frac{2}{15\Delta t}(u_{m-1}^{n+1}-u_{m-1}^{n-1})=$$
$$a\frac{u_{m+1}^{n+1}-u_{m-1}^{n+1}}{2\Delta x}-a\frac{u_{m+1}^{n}-u_{m-1}^{n}}{5\Delta x}+a\frac{u_{m+1}^{n-1}-u_{m-1}^{n-1}}{2\Delta x}. \tag{21}$$

特例 5 当 $\alpha_1=\frac{1}{5},\alpha_2=\frac{3}{4},\alpha_3=\frac{1}{20}$时为 3 层 9 点恒稳格式．

$$\frac{1}{60\Delta t}(13u_{m+1}^{n+1}-6u_{m+1}^{n}-7u_{m+1}^{n-1})+\frac{4}{60\Delta t}(13u_{m}^{n+1}-6u_{m}^{n}-7u_{m}^{n-1})+$$
$$\frac{1}{60\Delta t}(13u_{m-1}^{n+1}-6u_{m-1}^{n}-7u_{m-1}^{n-1})=$$
$$\frac{1}{5}a\frac{u_{m+1}^{n+1}-u_{m-1}^{n+1}}{\Delta x}+\frac{3}{4}a\frac{u_{m+1}^{n}-u_{m-1}^{n}}{\Delta x}+\frac{1}{20}a\frac{u_{m+1}^{n-1}-u_{m-1}^{n-1}}{\Delta x}. \tag{22}$$

格式(20)～(22)，都是本文所得新的 3 层恒稳差分格式．

4 数值例子

例如，考虑对流方程混合问题

$$\begin{cases}\dfrac{\partial u}{\partial t}-\dfrac{\partial u}{\partial x}=0, & 0<x<0,\ 0<t<T,\\ u(x,0)=\cos x & 0\le x\le 1,\\ u(1,t)=\cos(1+t) & t>0,\end{cases}$$

其精确解为 $u(x,t)=u(x,t)=\cos(x+t)$．此时，$a=1$．取 $\Delta t=0.015$，$\Delta x=0.1$，$r=0.15$，按上述格式(20)，(21)，(22)，计算到 $n=400$(即 $T=6$)并与精确解列表(表 1)比较．为了方便起

表 1 比较表($\Delta t=0.015$，$\Delta x=0.1$，$\gamma=0.15$，$n=400$)

x	精确解	格式(20)	格式(21)	格式(22)
0.0	0.960 170	0.960 170	0.960 170	0.960 170
0.1	0.983 268	0.983 262	0.983 244	0.983 271
0.2	0.996 542	0.996 548	0.996 562	0.996 540
0.3	0.999 859	0.999 855	0.999 846	0.999 860
0.4	0.993 185	0.993 194	0.993 219	0.993 182
0.5	0.976 588	0.976 585	0.976 575	0.976 589
0.6	0.950 233	0.950 241	0.950 266	0.950 230
0.7	0.914 383	0.914 376	0.914 356	0.914 385
0.8	0.869 397	0.869 402	0.869 415	0.869 396
0.9	0.815 725	0.815 712	0.815 677	0.815 729
SE[1]	–	0.000 007	0.000 026	0.000 002

[1] $SE=\{\frac{1}{10}\sum_{j=0}^{9}[u(x_j,t^N)-u_j^N]^2\}^{\frac{1}{2}}$为离散的平方误差

见，按精确值计算左边($x=0$)边界与第 1 层网格函数值的计算．数值例子表明，此理论分析的正确性．因此，本文的格式是有效的．

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A Group of Steady Schemes with High Accuracy for Solving Convective Equation

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Abstract　For solving convective equation $u_t=au_x$, a new group of implicit different schemes containing three parameters are constructed. They are three layers in general and two layers in special case. Their truncation error wile reach $O[(\Delta t)^2+(\Delta x)^4]$ at least. Under the conditions of $\alpha_1=\alpha_3$, $\alpha_2\leq 2\alpha_1$ or $\alpha_1\geq 0$, $\alpha_2\geq 0$, $\alpha_3\geq 0$, $\alpha_1>\alpha_3$, $\alpha_1+\alpha_2+\alpha_3=1$, $\alpha_2\leq\frac{1}{2}$, they are absolutely stable. Particalarly, a two layer steady difference scheme can be obtained in case parameter $\alpha_1=\alpha_2$, $\alpha_3=0$. All these schemes, with all steady ones with high accaracy in literatures included, can be solved by applying double sweeping method.

Keywords　convective equation, difference scheme, absolutely stable