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多重尺度法在流体动力学中的应用

陈 虹① 李大鸣②

(① 华侨大学土木工程系, 泉州 362011; ② 天津大学建筑工程学院, 天津 300072)

摘要 从Navier-Stokes(N-S)方程和质量守恒方程出发, 根据潮流、波浪和湍流等时间、空间尺度的不同特点, 引入摄动理论中的多重尺度法, 将方程中各物理量分解为与之相关的部分. 在不同尺度下作平均, 导出不同尺度下波流耦合作用连续方程、动量方程和泥沙方程, 用数学方法解释潮流波浪间的相互作用和影响. 分别讨论波浪、潮流作用下含沙量的垂向分布, 与现有理论公式及实测资料进行计算比较, 得到满意的结果, 有助于进一步研究流体动力学和泥沙运动学.

关键词 多重尺度法, 潮流, 波浪, 湍流

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1 基本方程及多重尺度法

流体运动的Navier-Stokes(N-S)方程为

$$\frac{\partial}{\partial t}u_i+\frac{\partial}{\partial x_j}(u_iu_j)=\frac{1}{\rho}\frac{\partial P}{\partial x_i}+X_i+\nu\frac{\partial^2 u_i}{\partial x_j^2},\tag{1}$$

$$\frac{\partial}{\partial x_j}u_j=0.\tag{2}$$

流体中另一介质(悬沙)的质量守恒方程(考虑紊动水流影响)为

$$\frac{\partial\Phi}{\partial t}+\frac{\partial}{\partial x_j}(u_j\Phi)+\frac{\partial}{\partial x_j}\left(\epsilon_j\frac{\partial\Phi}{\partial x_j}\right)=S_\Phi,\tag{3}$$

其中 u_i 为坐标 x_i 轴上的速度分量, P 为压力, X_i 为体积力(如重力), ρ 为流体密度, ν 为运动粘滞性系数, Φ 为介质浓度, ϵ 为扩散系数, S_Φ 为源汇项(如排沙洞等). 对一般状态, 此处 $S_\Phi=0$. 下标 $i=1,2,3$, $j=1,2,3$, 遵循张量求和约定.

流体运动遵循N-S方程, 但处理不同的运动形式要采用不同的处理方法, 甚至需要补充或简化某些作用项. 考虑水波中基本流(潮流)、波浪和湍流共同存在、相互作用的情况, 并注意到潮流的时间、空间尺度远大于波浪的时间、空间尺度, 而波浪的时间、空间尺度又大于湍流的时间、空间尺度. 本文引入奇异摄动理论中多重尺度法的两变量展开法[1~2], 对N-S方程和质量守恒方程中的不同尺度进行分离[3~4], 以便对不同尺度问题加以处理. 这样达到既能简便数值模拟, 又能反映实际物理过程的目的. 设

收稿日期 1999-05-05 作者简介 陈 虹(1969-), 男, 讲师
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$$u_i = u_i(x, t, \epsilon) = \sum_{k=0}^{\infty} \epsilon^k u_{ik}(T\zeta, T\eta, X\zeta, X\eta), \tag{4}$$

$$P = P(x, t, \epsilon) = \sum_{k=0}^{\infty} \epsilon^k P_k(T\zeta, T\eta, X\zeta, X\eta), \tag{5}$$

$$\Phi = \Phi(x, t, \epsilon) = \sum_{k=0}^{\infty} \epsilon^k \Phi_k(T\zeta, T\eta, X\zeta, X\eta), \tag{6}$$

$$X_i = \sum_{k=0}^{\infty} \epsilon^k X_{ik}, \tag{7}$$

其中 $T\zeta = t_0 + t_2 + \cdots, X\zeta_i = X_{0i} + X_{2i} + \cdots, T\eta = \epsilon t = t_1, X\eta_i = \epsilon x_i = x_{1i}$. $t_0 = t, X_{0i} = x_i$ 为大的时间及空间尺度(潮流的); $t_1 = \epsilon t, x_{1i} = \epsilon x_i$ 为中等的时间及空间尺度(波浪的); $t_2 = \omega_2 \epsilon^2 t, x_{2i} = \omega_2 \epsilon^2 x_i$ 为小的时间及空间尺度(湍流的); ϵ为小参数; ω_2 为待定系数. 则有

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial t_0} + \epsilon \frac{\partial}{\partial t_1} + \epsilon^2 \omega_2 \frac{\partial}{\partial t_2}; \quad \frac{\partial}{\partial x_i} = \frac{\partial}{\partial x_{0i}} + \epsilon \frac{\partial}{\partial x_{1i}} + \epsilon^2 \omega_2 \frac{\partial}{\partial x_{2i}}. \tag{8}$$

2 基本方程的分离

将式(4)～式(8)代入式(1)～式(3)中, 归纳 ϵ的同次幂项, 得

$$\frac{\partial}{\partial t_0} u_{i0} + \frac{\partial}{\partial x_{0j}} u_{i0} u_{j0} + \frac{1}{\rho} \frac{\partial}{\partial x_{0i}} P_0 - X_{i0} - \nu \frac{\partial^2}{\partial x_{0j}^2} u_{i0} = 0, \tag{9a}$$

$$\frac{\partial}{\partial x_{0j}} u_{0j} = 0, \tag{10a}$$

$$\frac{\partial}{\partial t_0} \Phi_0 + \frac{\partial}{\partial x_{0j}} \Phi_0 u_{j0} - \frac{\partial}{\partial x_{0j}} \epsilon_j \frac{\partial}{\partial x_{0j}} \Phi_0 = 0; \tag{11a}$$

$$\begin{aligned} &\frac{\partial}{\partial t_0} u_{i1} + \frac{\partial}{\partial t_1} u_{i0} + \frac{\partial}{\partial x_{0i}} (u_{i0} u_{j1} + u_{i1} u_{j0}) + \frac{\partial}{\partial x_{1j}} u_{i0} u_{j0} - X_{i1} + \\ &\frac{1}{\rho} \left(\frac{\partial}{\partial x_{0i}} P_1 + \frac{\partial}{\partial x_{1i}} P_0 \right) - \nu \frac{\partial^2}{\partial x_{0j}^2} u_{i1} - 2\nu \frac{\partial}{\partial x_{0j}} \frac{\partial}{\partial x_{1j}} = 0, \end{aligned} \tag{9b}$$

$$\frac{\partial}{\partial x_{0j}} u_{j1} + \frac{\partial}{\partial x_{1j}} u_{j0} = 0, \tag{10b}$$

$$\begin{aligned} &\frac{\partial}{\partial t_1} \Phi_0 + \frac{\partial}{\partial t_0} \Phi_1 + \frac{\partial}{\partial x_{0j}} (\Phi_1 u_{j0} + \Phi_0 u_{j1}) + \frac{\partial}{\partial x_{1i}} (\Phi_0 u_{j0}) + \\ &\frac{\partial}{\partial x_{0j}} \epsilon_j \frac{\partial}{\partial x_{0j}} \Phi_1 + 2 \frac{\partial}{\partial x_{1j}} \epsilon_j \frac{\partial}{\partial x_{0j}} \Phi_0 = 0; \end{aligned} \tag{11b}$$

$$\begin{aligned} &\frac{\partial}{\partial t_0} u_{i2} + \frac{\partial}{\partial t_1} u_{i1} + \omega_2 \frac{\partial}{\partial t_2} u_{i0} + \frac{\partial}{\partial x_{0j}} (u_{i0} u_{j2} + u_{i1} u_{j1} + u_{i2} u_{j0}) + \\ &\frac{\partial}{\partial x_{1j}} (u_{i0} u_{j1} + u_{i1} u_{j0}) + \omega_2 \frac{\partial}{\partial x_{2j}} u_{i0} u_{j0} + \frac{1}{\rho} \Big(\frac{\partial}{\partial x_{0i}} P_2 + \\ &\frac{\partial}{\partial x_{1j}} P_1 + \omega_2 \frac{\partial}{\partial x_{2j}} P_0 \Big) - X_{i2} - \nu \frac{\partial^2}{\partial x_{0j}^2} u_{i2} \\ &- 2\nu \frac{\partial}{\partial x_{1j}} \frac{\partial}{\partial x_{0j}} u_{i1} - 2\nu \left(\omega_2 \frac{\partial}{\partial x_{0j}} \frac{\partial}{\partial x_{2j}} + \frac{\partial^2}{\partial x_{1j}^2} \right) u_{i0} = 0, \end{aligned} \tag{9c}$$

$$\frac{\partial}{\partial x_{0j}} u_{j2} + \frac{\partial}{\partial x_{1j}} u_{j1} + \omega_2 \frac{\partial}{\partial x_{2j}} u_{j0} = 0, \tag{10c}$$

$$\frac{\partial}{\partial t_0}\Phi_2+\frac{\partial}{\partial t_1}\Phi_1+\omega_2\frac{\partial}{\partial t_2}\Phi_0+\frac{\partial}{\partial x_{0j}}(\Phi_2 u_{j0}+\Phi_1 u_{j1}+\Phi_0 u_{j2})+$$
$$\frac{\partial}{\partial x_{1j}}(\Phi_1 u_{j0}+\Phi_0 u_{j1})+\omega_2\frac{\partial}{\partial x_{2j}}(\Phi_0 u_{j0})+\frac{\partial}{\partial x_{0j}}\epsilon_j\frac{\partial}{\partial x_{0j}}\Phi_2+2\frac{\partial}{\partial x_{1j}}\epsilon_j\frac{\partial}{\partial x_{0j}}\Phi_1+$$
$$2\frac{\partial}{\partial x_{1j}}\epsilon_j\frac{\partial}{\partial x_{1j}}\Phi_0+2\omega_2\frac{\partial}{\partial x_{2j}}\epsilon_j\frac{\partial}{\partial x_{0j}}\Phi_0=0. \tag{11c}$$

式中 标 a 表示 ϵ 的零阶项, 标号 b 表示 ϵ 的一阶项, 标号 c 表示 ϵ 的二阶项(下同). 设 ϵ 的各阶速度、压力和含沙量分量受潮流、波浪及湍动作用, 具有相同的量级. 对它们作类似 Reynolds 分解形式的分解, 并考虑到大尺度变量在小尺度下变化较小, 取前三项则得

$$u_{ik}(t_0,t_1,t_2,\cdots,x_0,x_1,x_2,\cdots)=\sum u_{1ik}(t_0,t_1,t_2,\cdots,x_0,x_1,x_2,\cdots)\approx$$
$$u_{0ik}(t_0,x_0)+u_{1ik}(t_0,t_1,x_0,x_1)+u_{2ik}(t_0,t_1,t_2,x_0,x_1,x_2), \tag{12}$$
$$P_k(t_0,t_1,t_2,\cdots,x_0,x_1,x_2,\cdots)=\sum P_{1k}(t_0,t_1,t_2,\cdots,x_0,x_1,x_2,\cdots)\approx$$
$$P_{0k}(t_0,x_0)+P_{1k}(t_0,t_1,x_0,x_1)+P_{2k}(t_0,t_1,t_2,x_0,x_1,x_2), \tag{13}$$
$$\Phi_k(t_0,t_1,t_2,\cdots,x_0,x_1,x_2,\cdots)=\sum\Phi_{1k}(t_0,t_1,t_2\cdots,x_0,x_1,x_2\cdots)\approx$$
$$\Phi_{0k}(t_0,x_0)+\Phi_{1k}(t_0,t_1,x_0,x_1)+\Phi_{2k}(t_0,t_1,t_2,x_0,x_1,x_2). \tag{14}$$

在式(12)～式(14)中, u_{1ik}, P_{1k}, Φ_{1k_2} 是以 T_1 为周期的周期函数, u_{2ik}, P_{2k}, Φ_{2k} 是以比 T_2 尺度更小尺度的随机函数, 且 $t_0\geq T_1>t_1\geq T_2>t_2$. 将式(12)～式(14)代入式(9)～式(11)中, 并考虑式中 ϵ 的零阶近似无法以解析式给出, 不适宜应用 Green 公式继续求解[5]. 在此应用 Frodholin 择一定理[6], 即选择 '强迫力项' 为零的近似来代入式中. 这也是多重尺度展开法特点的解释, 即为保证展开式的一致有效性, 设法去除掉长期项影响[7]. 考虑到各变量的生成原因(作用项)及尺度表达不同, 可将其分离开以便于理解和分别处理. 由此分离不同尺度后, 并考虑到小尺度变量可由高阶方程中消去长期项影响的方程而决定, 而大尺度变量在小尺度下可视为常量. 得到一组近似方程为

$$\frac{\partial}{\partial t_0}(u_{0i0}+u_{1i0}+u_{2i0})+\frac{\partial}{\partial x_{0j}}(u_{0i0}+u_{1i0}+u_{2i0})(u_{0j0}+u_{1j0}+u_{2j0})+$$
$$\frac{1}{\rho}\left(\frac{\partial}{\partial x_{0j}}(P_{00}+P_{10}+P_{20})-X_{i0}-\nu\frac{\partial^2}{\partial x_{0j}^2}(u_{0i0}+u_{1i0}+u_{2i0})=0,\right. \tag{15a}$$

$$\frac{\partial}{\partial x_{0j}}(u_{0j0}+u_{1j0}+u_{2j0})=0, \tag{16a}$$

$$\frac{\partial}{\partial t_0}(\Phi_{00}+\Phi_{10}+\Phi_{20})+\frac{\partial}{\partial x_{0j}}(\Phi_{00}+\Phi_{10}+\Phi_{20})(u_{0j0}+$$
$$u_{1j0}+u_{2j0})-\frac{\partial}{\partial x_{0j}}\epsilon_j\frac{\partial}{\partial x_{0j}}(\Phi_{00}+\Phi_{10}+\Phi_{20})=0; \tag{17a}$$

$$\frac{\partial}{\partial t_1}(u_{1i0}+u_{2i0})+\frac{\partial}{\partial x_{1j}}(u_{0i0}+u_{1i0}+u_{2i0})(u_{0j0}+u_{1j0}+u_{2j0})+$$
$$\frac{1}{\rho}\frac{\partial}{\partial x_{1i}}(P_{10}+P_{20})-X_{i1}-2\nu\frac{\partial}{\partial x_{0j}}\frac{\partial}{\partial x_{1j}}(u_{1i0}+u_{2i0})=0, \tag{18b}$$

$$\frac{\partial}{\partial x_{1j}}(u_{1j0}+u_{2j0})=0, \tag{19b}$$

$$\frac{\partial}{\partial t_1}(\Phi_{10}+\Phi_{20})+\frac{\partial}{\partial x_{1j}}(\Phi_{00}+\Phi_{10}+\Phi_{20})(u_{0j0}+u_{1j0}+u_{2j0})+$$

$$2\frac{\partial}{\partial x_{0j}}\epsilon_j\frac{\partial}{\partial x_{1j}}(\Phi_{00}+\Phi_{10}+\Phi_{20})=0; \tag{20b}$$

$$\frac{\partial}{\partial t_2}(u_{2i0})+\frac{\partial}{\partial x_{2j}}(u_{0i0}+u_{1i0}+u_{2i0})(u_{0j0}+u_{1j0}+u_{2j0})+$$
$$\frac{1}{\rho}\frac{\partial}{\partial x_{2j}}(P_{20})-X_{i2}-2\nu\frac{\partial}{\partial x_{0j}}\frac{\partial}{\partial x_{2j}}(u_{2i0})=0, \tag{21c}$$

$$\frac{\partial}{\partial x_{2j}}(u_{2j0})=0, \tag{22c}$$

$$\frac{\partial}{\partial t_2}(\Phi_{20})+\frac{\partial}{\partial x_{2j}}(\Phi_{00}+\Phi_{10}+\Phi_{20})(u_{0j0}+u_{1j0}+u_{2j0})-2\frac{\partial}{\partial x_{2j}}\epsilon_j\frac{\partial}{\partial x_{0j}}(\Phi_{20})=0. \tag{23c}$$

由于$t_0\geq T_1>t_1\geq T_2>t_2$,对方程(15)～(17)的大尺度方程可以先在$T_2$尺度水平上平均,再在$T_1$尺度水平上平均,取到$\epsilon$零阶项的量级.对方程(18)～(20)中的波浪尺度方程,可在T_2尺度水平上平均,取到ϵ零阶项的量级.由此,可得到不同尺度的联立方程为

$$\frac{\partial}{\partial t_0}(u_{0i0})+\frac{\partial}{\partial x_{0j}}(u_{0i0}u_{0j0})+\frac{\partial}{\partial x_{0j}}\overline{(u_{1i0}u_{1j0})}+$$
$$\frac{1}{\rho}\frac{\partial}{\partial x_{0i}}(P_{00})-X_{i0}-\nu\frac{\partial^2}{\partial x_{0j}^2}(u_{0i0})=0, \tag{24a}$$

$$\frac{\partial}{\partial x_{0j}}(u_{0j0})=0, \tag{25a}$$

$$\frac{\partial}{\partial t_0}(\Phi_{00})+\frac{\partial}{\partial x_{0j}}(\Phi_{00}\Phi_{0j0})+\frac{\partial}{\partial x_{0j}}\overline{(\Phi_{10}u_{1j0})}-\frac{\partial}{\partial x_{0j}}\epsilon_j\frac{\partial}{\partial x_{0j}}(\Phi_{00})=0; \tag{26a}$$

$$\frac{\partial}{\partial t_1}(u_{1i0})+\frac{\partial}{\partial x_{1j}}(u_{0i0}+u_{1i0})(u_{0j0}+u_{1j0})+\frac{\partial}{\partial x_{1j}}\overline{(u_{2i0}u_{2j0})}+$$
$$\frac{1}{\rho}\frac{\partial}{\partial x_{1i}}(P_{10})-X_{i1}-2\nu\frac{\partial}{\partial x_{0j}}\frac{\partial}{\partial x_{1j}}(u_{1i0})=0, \tag{27b}$$

$$\frac{\partial}{\partial x_{1j}}(u_{1j0})=0, \tag{28b}$$

$$\frac{\partial}{\partial t_1}(\Phi_{10})+\frac{\partial}{\partial x_{1j}}(\Phi_{00}+\Phi_{10})(u_{0j0}+u_{1j0})+\frac{\partial}{\partial x_{1j}}\overline{(\Phi_{20}u_{2j0})}+2\frac{\partial}{\partial x_{0j}}\epsilon_j\frac{\partial}{\partial x_{1j}}(\Phi_{10})=0; \tag{29b}$$

$$\frac{\partial}{\partial t_2}(u_{2i0})+\frac{\partial}{\partial x_{2j}}(u_{0i0}+u_{1i0}+u_{2i0})(u_{0j0}+u_{1j0}+u_{2j0})+$$
$$\frac{1}{\rho}\frac{\partial}{\partial x_{2j}}P_{20}-X_{i2}-2\nu\frac{\partial}{\partial x_{0j}}\frac{\partial}{\partial x_{2j}}(u_{2i0})=0, \tag{30c}$$

$$\frac{\partial}{\partial x_{2j}}(u_{2j0})=0, \tag{31c}$$

$$\frac{\partial}{\partial t_2}(\Phi_{20})+\frac{\partial}{\partial x_{2j}}(\Phi_{00}+\Phi_{10}+\Phi_{20})(u_{0j0}+u_{1j0}+u_{2j0})+2\frac{\partial}{\partial x_{0j}}\epsilon_j\frac{\partial}{\partial x_{2j}}=0. \tag{32c}$$

式(24a)～式(26a),式(27b)～式(29b)和式(30c)～式(32c),它们分别为带有波浪作用下的大尺度潮流、潮流,湍流作用下的小尺度波浪、潮流,波浪作用下的湍流所满足的动量方程、连续方程和质量守恒方程.

3　含沙量垂向分布验证

为验证上述多重尺度法展开的合理性, 在此仅对质量守恒方程作一比较. 先考虑式(29b)垂向($j=3$)二维状态, 即波浪作用下含沙量垂向分布的情况. 并注意到 u_{030}为大尺度下的垂向速度, 平衡状态时式(29b)可化为

$$\frac{\partial}{\partial x_{13}}\left[\Phi_{10}(u_{030}+u_{130})+2\epsilon_3\frac{\partial}{\partial x_{03}}(\Phi_{10})\right]=0. \tag{33}$$

积分上式, 并引入清水条件, 当含沙量为垂向变化率为零时, 含沙量为零. 则有

$$\Phi_{10}(u_{030}+u_{130})+2\epsilon_3\frac{\partial}{\partial x_{03}}(\Phi_{10})=0. \tag{34}$$

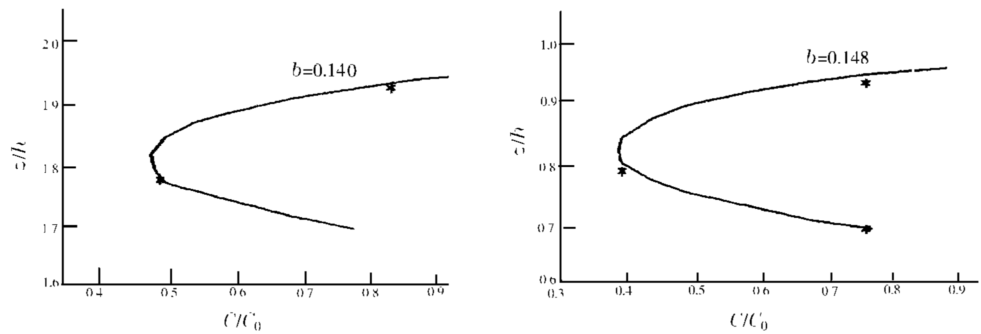
取

$$u_{130}=bu_0\operatorname{sh}k(h-z)=\frac{b\pi H}{T\operatorname{sh}\left(\frac{2\pi h}{L}\right)}\operatorname{sh}k(h-z) \tag{35}$$

为波浪质点的垂向最大速度, b 为一系数. 式(34)中 $u_{030}=-w$ 为悬移质静水沉速, $\epsilon_3=\epsilon_z=$ const 为垂向扩散系数. 积分式(35), 得

$$\frac{\Phi_{10}}{\Phi_a}=\exp\left\{\frac{1}{2\epsilon_z}\left[w(z-a)+bu_0\operatorname{ch}k(h-a)-bu_0\operatorname{ch}k(h-z)\right]\right\}, \tag{36}$$

Φ_a为床面层上含沙量. 文[8]曾假定垂向扩散系数 $\epsilon_z=Kl^2\left(\frac{\mathrm{d}u}{\mathrm{d}y}\right)$. 文[9]假定垂向扩散系数 $\epsilon_z=\frac{b\cdot gTH\operatorname{th}k(h-z)}{4\pi\operatorname{ch}kh}$, 沉速为 $W=W_0+R_*\frac{\partial\epsilon}{\partial z}$均导出与式(36)类似的结果. 同时将文[10]与式(36)比较(图1), 计算结果满意.



(a) F 组　　(b) G 组

图 1　式(36)与实测资料比较

再考虑式(26a)垂向($j=3$)的二维状态, 即潮流作用下含沙量垂向分布的情况. 在平衡状态时, 式(26b)又可化为

$$\frac{\partial}{\partial x_{03}}\left[\Phi_{00}u_{030}+\overline{(\Phi_{10}u_{130})}-\epsilon_j\frac{\partial}{\partial x_{03}}\Phi_{00}\right]=0. \tag{37}$$

取 $u_{030}=\omega$, $\epsilon_3=\epsilon_z=$ const, 并设$\overline{(\Phi_{10}u_{130})}=\frac{1}{T}\int_0^T\Phi_{10}u_{130}\mathrm{d}t$, $\Phi_{00}=\Phi_2$, 则

$$\Phi_2=\Phi_a e^{-\frac{w(z-a)}{\epsilon_z}}\left(\int_a^z\frac{1}{2}\Phi_{10}u_{130}e^{-wz}\mathrm{d}z+1\right) \tag{39}$$

当无波浪作用时, $u^{130} = 0$, $\Phi_z = \Phi_\alpha e^{-\frac{w(z-\alpha)}{\epsilon_z}}$. 与文献[1]中一致.

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Application of Multiple Scale Method to the Study of Hydrodynamics

Chen Hong① Li Daming②

(① Dept. of Civil. Eng., Huaqiao Univ. 362011, Quanzhou;
② Inst. of Civil. Eng., Tianjin Univ, 300072, Tianjin)

Abstract Starting from Navier-Stokes equation and equation of conservation of mass, multiple scale method from perturbation theory is drawn into the study of hydrodynamics in line with the characteristics of tidal current, wave, turbulence and other temporal and spatial sacles. By which all the physical quantities in N-S equation are decomposed into parts correlated with it and the mean is taken under different scales. A continous equation of wave and tide coupling under different scales and a momentum equation and a silt equation are derived for mathematical explaination and discussion. The interaction and the mutual influence of wave and tide are mathematically explained; and the vertical distribution of sediment concentration under the action of wave and tide is discussed respectively. As compared with theoretical formula and field data now available, the authors' results are satisfactory. They are conducive to the further study of hydrodynamics and silt transportation.

Keywords multiple scale method, tidal current, wave, turbulence