场论说给出包含互感的四种元件串并联公式*

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摘要　应用电磁场能量转换与守恒定律于正弦稳态线性交流网络中, 得出能量守恒定律的新表示式. 利用式中统一体现的电阻、电容、自感和互感 4 种元件能量形式的优点, 以及网络现代场论关于元件复阻抗率和积分形式的复阻抗等概念, 分别给出 4 种元件具体的积分形式复阻抗. 通过复阻抗与电压、电流的关系, 与电路的串并联联系起来, 导出包含互感元件的 4 种元件串联后再串并联的复阻抗公式, 并验证此公式的正确性.

关键词　互感, 电路元件, 串并联公式

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一般电路理论[1,2]在线性交流网络中只能分别讨论电阻、电容、自感 3 种元件的串并联公式和互感串并联公式, 而且往往对互感的串并联单独讨论. 可是从网络现代场论[3~5]看来, 电阻、电容、自感和互感这 4 种元件的能量形式都遵循能量转移与守恒定律. 对复阻抗率积分得到元件的复阻抗, 再通过复阻抗与电压、电流的关系, 与电路的串并联联系起来, 从而导出在线性交流网络中 4 种元件的串并联公式. 为统一 4 种元件的串并联公式提供了理论依据.

1　能量转换与守恒定律

把复数形式的能量守恒定律应用于含 N 个节点, B 条支路的线性交流网络[3,6]为

$$\sum_{i=1}^{B}\int \hat{j}_i^{*}\hat{\boldsymbol{K}}\,\mathrm{d}\boldsymbol{l}_i=\sum_{i=1}^{B}\int \rho_i^{(0)(R+r)}\hat{j}_i^{*}\hat{\boldsymbol{j}}_i\,\mathrm{d}\boldsymbol{l}_i+\frac{\mathrm{i}\omega}{S'}\sum_{i=1}^{B}\int\left(\mu^{(0)}\hat{\boldsymbol{H}}_i\hat{\boldsymbol{H}}_i^{*}-\epsilon^{(0)}\hat{\boldsymbol{E}}_i\hat{\boldsymbol{E}}_i^{*}\right)\mathrm{d}\Omega_i. \tag{1}$$

可以将式(1)右边的后一项积分也表成右边第一项的形式[3], 即

$$\sum_{i=1}^{B}\hat{j}_i^{*}\left(\int\hat{\boldsymbol{K}}_i\,\mathrm{d}\boldsymbol{l}_i-\int\hat{z}_i\hat{\boldsymbol{j}}_i\,\mathrm{d}\boldsymbol{l}_i\right)-\sum_{g=1}^{B}\hat{j}_g^{*}\sum_{i=1}^{B}\int\hat{z}_{gi}^{(M)}\hat{\boldsymbol{j}}_i\,\mathrm{d}\boldsymbol{l}_i=0, \tag{2}$$

式中 $\hat{z}_i=\rho_i^{(0)(R+r)}+\hat{z}_i^{(C)}+\hat{z}_i^{(L)}$ 称为第 i 支路电阻、电容和电感 3 种元件的复合复阻抗率, $\hat{z}_{gi}^{(M)}=\dfrac{\mathrm{i}\omega\mu^{(0)}S'}{4\pi}\displaystyle\int\frac{\cos(\boldsymbol{j}_g,\boldsymbol{j}_i)}{h_{gi}}\mathrm{d}l_g$ 则表示 g 对 i 支路互感复阻抗率.

2　4 种元件先串联再串联的公式

电路的组成如图 1 所示. 图中表示两组 4 种元件串联后又相串联的情况. 它们的参量分

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别为R_1, C_1, L_1, M_{12}和R_2, C_2, L_2, M_{21}. 图 1 所示的电路可看成有 3 条支路, 其中第 1 条支路是R_1, C_1, L_1和M_{12}串联的支路; 第 2 条支路是R_2, C_2, L_2, M_{21}串联的支路; 第 3 条支路仅有电源$\hat{\boldsymbol{K}}$分布. 把式(2)应用于图 1 得

$$-\hat{j}_1^*\int\hat{z}_1\hat{\boldsymbol{j}}_1\,\mathrm{d}\boldsymbol{l}_1-\hat{j}_2^*\int\hat{z}_{21}^{(M)}\hat{\boldsymbol{j}}_1\,\mathrm{d}\boldsymbol{l}_1-\hat{j}_2^*\int\hat{z}_2\hat{\boldsymbol{j}}_2\,\mathrm{d}\boldsymbol{l}_2-\hat{j}_2^*\int\hat{z}_{12}^{(M)}\hat{\boldsymbol{j}}_2\,\mathrm{d}\boldsymbol{l}_2+\hat{j}_3^*\oint\hat{\boldsymbol{K}}\,\mathrm{d}\boldsymbol{l}_3=0, \tag{4}$$

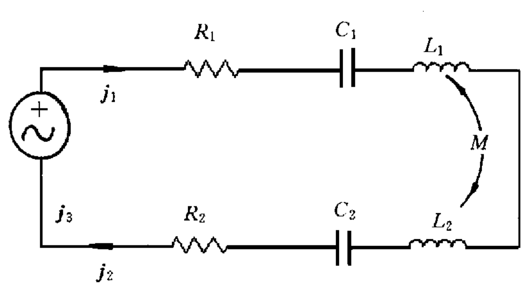

图 1　4 种元件先串联再相串联的电路

式中各项已标定电流方向与支路方向一致. 由串联电路的电流关系有$\hat{j}_1=\hat{j}_2=\hat{j}_3$, 并令他们都等于$\hat{j}$, 然后代入式(4)得

$$\int\hat{z}_1\hat{\boldsymbol{j}}\,\mathrm{d}\boldsymbol{l}_1+\int\hat{z}_{21}^{(M)}\hat{\boldsymbol{j}}\,\mathrm{d}\boldsymbol{l}_1+\int\hat{z}_2\hat{\boldsymbol{j}}\,\mathrm{d}\boldsymbol{l}_2-\int\hat{z}_{12}^{(M)}\hat{\boldsymbol{j}}\,\mathrm{d}\boldsymbol{l}_2=\oint\hat{\boldsymbol{K}}\,\mathrm{d}\boldsymbol{l}_3.$$

把电流密度$\hat{j}$变换为电流I, 上式改写为

$$I\int\hat{z}_1\frac{\mathrm{d}l_1}{S'}+I\int\hat{z}_{21}^{(M)}\frac{\mathrm{d}l_1}{S'}+I\int\hat{z}_2\frac{\mathrm{d}l_2}{S'}+I\int\hat{z}_{12}^{(M)}\frac{\mathrm{d}l_2}{S'}=\oint\hat{\boldsymbol{K}}\,\mathrm{d}\boldsymbol{l}_3, \tag{5}$$

式中$\hat{z}_1=\rho_1^{(0)(R)}+\hat{z}_1^{(C)}+\hat{z}_1^{(L)}$称为第 1 支路的复合复阻抗率, $\hat{z}_2=\rho_2^{(0)(R)}+\hat{z}_2^{(C)}+\hat{z}_2^{(L)}$称为第 2 支路的复合复阻抗率, $\hat{z}_{12}^{(M)}$和$\hat{z}_{21}^{(M)}$称为互感元件的复阻抗率. 若令

$$\hat{Z}_1=\int\hat{z}_1\frac{\mathrm{d}l_1}{S'},\quad \hat{Z}_2=\int\hat{z}_2\frac{\mathrm{d}l_2}{S'},\quad \hat{Z}_{12}^{(M)}=\int\hat{z}_{12}^{(M)}\frac{\mathrm{d}l_2}{S'},\quad \hat{Z}_{21}^{(M)}=\int\hat{z}_{21}^{(M)}\frac{\mathrm{d}l_1}{S'}, \tag{6}$$

则式(5)可写为

$$\hat{I}(\hat{Z}_1+\hat{Z}_2+2\hat{Z}^{(M)})=\oint\hat{\boldsymbol{K}}\,\mathrm{d}\boldsymbol{l}_3, \tag{7}$$

式中$\hat{Z}_1$, $\hat{Z}_2$和$\hat{Z}^{(M)}$分别称为第 1 条支路的复合复阻抗、第 2 条支路的复合复阻抗和互感复阻抗[3]. 其中不难证明$\hat{Z}_{12}^{(M)}=\hat{Z}_{21}^{(M)}$. 因$\hat{Z}_{12}^{(M)}=\int_2\hat{z}_{12}^{(M)}\frac{\mathrm{d}l_2}{S'}$, 故把式(3)代入可得

$$\hat{Z}_{12}^{(M)}=\int_2\left[\frac{\mathrm{i}\omega\mu^{(0)}S'}{4\pi}\int_1\frac{\cos(\boldsymbol{j}_1,\boldsymbol{j}_2)}{h_{12}}\mathrm{d}l_1\right]\frac{\mathrm{d}l_2}{S'}=\int_1\left[\frac{\mathrm{i}\omega\mu^{(0)}S'}{4\pi}\int_2\frac{\cos(\boldsymbol{j}_2,\boldsymbol{j}_1)}{h_{21}}\mathrm{d}l_2\right]\frac{\mathrm{d}l_1}{S'}=\int_1\hat{z}_{21}^{(M)}\frac{\mathrm{d}l_1}{S'}=\hat{Z}_{21}^{(M)}=Z^{(M)}. \tag{8}$$

若用$\hat{Z}$表示 4 种元件先串联再相串联的电路的等效复阻抗, 那么由式(7)可得两组 4 种元件串联在一起又彼此串联的等效复阻抗为

$$\hat{Z}=\hat{Z}_1+\hat{Z}_2+2\hat{Z}^{(M)}. \tag{9}$$

按复阻抗与各元件参量的关系

$$\hat{Z}^{(R)}=R,\quad \hat{Z}^{(C)}=\frac{1}{\mathrm{i}\omega C},\quad \hat{Z}^{(L)}=\mathrm{i}\omega L,\quad \hat{Z}^{(M)}=\mathrm{i}\omega M. \tag{10}$$

将式(9)改写为

$$\hat{Z}=R_1+R_2+\mathrm{i}\omega(L_1+L_2)+\frac{1}{\mathrm{i}\omega}\left(\frac{1}{C_1}+\frac{1}{C_2}\right)+2\hat{Z}^{(M)}, \tag{11}$$

其中的互感项由式(8)和式(10)可得

$$\hat{Z}^{(M)} = \cos(\boldsymbol{j}_2,\boldsymbol{j}_1)\ \hat{Z}^{(M)} = \mathrm{i}\omega\cos(\boldsymbol{j}_2,\boldsymbol{j}_1)\ M = \pm\,\mathrm{i}\omega M\ . \tag{12}$$

当 $\boldsymbol{j}_1$ 和 $\boldsymbol{j}_2$ 是从线圈的同名端流入时，式(12)取正号；当从异名端流入时，取负号．把上式代入式(11)，可得到通过 4 种元件参量表出的复阻抗公式为

$$\hat{Z} = R_1 + R_2 + \mathrm{i}\omega(L_1 + L_2) + \frac{1}{\mathrm{i}\omega}\left(\frac{1}{C_1} + \frac{1}{C_2}\right) \pm \mathrm{i}\omega M\ . \tag{13}$$

当只有电阻元件，得 2 个电阻的串联公式，$R = R_1 + R_2$；当只有电容元件，得 2 个电容的串联公式，$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$；当只有电感元件，得 2 个电感的串联公式，$L = L_1 + L_2$；当只有电感元件，则得 2 个互感串联的公式，$L = L_1 + L_2 \pm M$．

3　4 种元件先串联再相并联的公式

图 2 表示两组 4 种元件先串联再相并联．它们的参量分别为 R_1, C_1, L_1, M_{12} 和 R_2, C_2, L_2, M_{21}．图 2 所示的电路可看成有 3 条支路，其中第 1 条支路是 R_1, C_1, L_1 和 M_{12} 串联的支路；第 2 条支路是 R_2, C_2, L_2, M_{21} 串联支路；第 3 条支路仅有电源 $\hat{\boldsymbol{K}}$ 分布．把式(2)应用于图 2 得

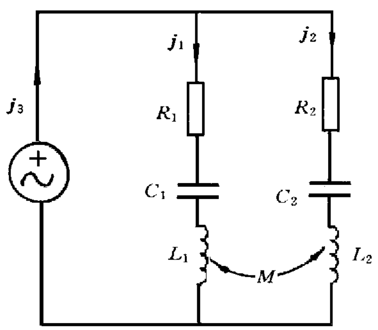

图 2　4 种元件串联先再相并联的电路

$$\hat{j}_1^*\int \hat{z}_1\hat{\boldsymbol{j}}_1\ \mathrm{d}\boldsymbol{l}_1 + \hat{j}_1^*\int \hat{z}_{12}^{(M)}\hat{\boldsymbol{j}}_2\ \mathrm{d}\boldsymbol{l}_2 + \hat{j}_2^*\int \hat{z}_2\hat{\boldsymbol{j}}_2\ \mathrm{d}\boldsymbol{l}_2 + \hat{j}_2^*\int \hat{z}_{21}^{(M)}\hat{\boldsymbol{j}}_1\ \mathrm{d}\boldsymbol{l}_1 + \hat{j}_3^*\int \hat{\boldsymbol{K}}\ \mathrm{d}\boldsymbol{l}_3, \tag{14}$$

式中 $\hat{z}_1 = = \rho_1^{(0)(R)} + \hat{z}_1^{(C)} + \hat{z}_1^{(L)}$ 称为第 1 支的复合复阻抗率；$\hat{z}_2 = = \rho_2^{(0)(R)} + \hat{z}_2^{(C)} + \hat{z}_2^{(L)}$ 称为第 2 支的复合复阻抗率；$\hat{z}_{12}^{(M)}$ 和 $\hat{z}_{21}^{(M)}$ 称为互感元件的复阻抗率．上式中各项已标定电流方向与支路方向一致．由并联电路的电流关系有

$$\hat{\boldsymbol{j}}_3 = \hat{\boldsymbol{j}}_1 + \hat{\boldsymbol{j}}_2. \tag{15}$$

将式(15)代入式(14)得

$$\hat{j}_1^*\left(\int \hat{z}_1\hat{\boldsymbol{j}}_1\ \mathrm{d}\boldsymbol{l}_1 + \int \hat{z}_{12}^{(M)}\hat{\boldsymbol{j}}_2\ \mathrm{d}\boldsymbol{l}_2 - \int \hat{\boldsymbol{K}}\ \mathrm{d}\boldsymbol{l}_3\right) =$$

$$\hat{j}_2^*\left(\int \hat{z}_2\ \hat{\boldsymbol{j}}_2\ \mathrm{d}\boldsymbol{l}_2 + \int \hat{z}_{21}^{(M)}\hat{\boldsymbol{j}}_1\ \mathrm{d}\boldsymbol{l}_1 - \int \hat{\boldsymbol{K}}\ \mathrm{d}\boldsymbol{l}_3\right), \tag{16}$$

令 $\int \hat{z}_1\hat{\boldsymbol{j}}_1\cdot\mathrm{d}\boldsymbol{l}_1 + \int \hat{z}_{12}^{(M)}\hat{\boldsymbol{j}}_2\cdot\mathrm{d}\boldsymbol{l}_2 = \int \hat{\boldsymbol{K}}\cdot\mathrm{d}\boldsymbol{l}_3$，$\int \hat{z}_2\cdot\hat{\boldsymbol{j}}_2\cdot\mathrm{d}\boldsymbol{l}_2 + \int \hat{z}_{21}^{(M)}\hat{\boldsymbol{j}}_1\cdot\mathrm{d}\boldsymbol{l}_1 = \int \hat{\boldsymbol{K}}\cdot\mathrm{d}\boldsymbol{l}_3$．若 $\hat{\boldsymbol{j}}_1, \hat{\boldsymbol{j}}_2$ 用电流强度 $\hat{I}_1, \hat{I}_2$ 表示，则可写成如下形式：

$$\hat{I}_1\int \hat{z}_1\frac{\mathrm{d}\boldsymbol{l}_1}{S} + \hat{I}_2\int \hat{z}_{12}^{(M)}\frac{\mathrm{d}\boldsymbol{l}_2}{S} = \int \hat{\boldsymbol{K}}\ \mathrm{d}\boldsymbol{l}_3,\qquad \hat{I}_2\int \hat{z}_2\frac{\mathrm{d}\boldsymbol{l}_2}{S} + \hat{I}_1\int \hat{z}_{21}^{(M)}\frac{\mathrm{d}\boldsymbol{l}_1}{S} = \int \hat{\boldsymbol{K}}\ \mathrm{d}\boldsymbol{l}_3. \tag{17}$$

令 $\hat{Z}_1 = \int \hat{z}_1\frac{\mathrm{d}\boldsymbol{l}_1}{S}$；$\hat{Z}_2 = \int \hat{z}_2\frac{\mathrm{d}\boldsymbol{l}_2}{S}$；$\hat{z}_{12}^{(M)} = \int \hat{z}_{12}^{(M)}\frac{\mathrm{d}\boldsymbol{l}_2}{S}$；$\hat{z}_{21}^{(M)} = \int \hat{z}_{21}^{(M)}\frac{\mathrm{d}\boldsymbol{l}_1}{S}$，并将其代入式(17)．得

$$\hat{I}_1 = \frac{(\hat{Z}_2 - \hat{Z}^{(M)})\int \hat{\boldsymbol{K}}\ \mathrm{d}\boldsymbol{l}_3}{\hat{Z}_1\hat{Z}_2 - (\hat{Z}^{(M)})^2},\qquad \hat{I}_2 = \frac{(\hat{Z}_1 - \hat{Z}^{(M)})\int \hat{\boldsymbol{K}}\ \mathrm{d}\boldsymbol{l}_3}{\hat{Z}_1\hat{Z}_2 - (\hat{Z}^{(M)})^2},$$

式中$\hat{Z}_1$, $\hat{Z}_2$和$\hat{Z}^{(M)}$分别称为第1支条路的复合复阻抗、第2条支路的复合复阻抗和互感阻抗,由式(8)的证明有$\hat{Z}_{12}^{(M)}=\hat{Z}_{21}^{(M)}=\hat{Z}^{(M)}$. 再把$\hat{I}_1$, $\hat{I}_2$代入$\hat{I}_3=\hat{I}_1+\hat{I}_2$得

$$\hat{I}_3=[(\hat{Z}_1+\hat{Z}_2-2\hat{Z}^{(M)})\oint\hat{K}\,\mathrm{d}l_3]/[\hat{Z}_1\hat{Z}_2-(\hat{Z}^{(M)})^2].$$

若用$\hat{Z}$表示4种元件先串联再相并联的并联电路等效复阻抗. 则由上式可得到4种元件先串联再相并联的等效复阻抗为

$$\hat{Z}=[\hat{Z}_1\hat{Z}_2-(\hat{Z}^{(M)})^2]/(\hat{Z}_1+\hat{Z}_2-2\hat{Z}^{(M)}). \tag{18}$$

按复阻抗与元件参量之间的关系, 利用式(10)和式(12), 并通过式(18)得到4种元件先串联再相并联的公式为

$$\hat{Z}=\frac{\left(R_1+\frac{1}{\mathrm{i}\omega C_1}+\mathrm{i}\omega L_1\right)\left(R_2+\frac{1}{\mathrm{i}\omega C_2}+\mathrm{i}\omega L_2\right)-\omega^2M^2}{R_1+R_2+\mathrm{i}\omega(L_1+L_2)+\frac{1}{\mathrm{i}\omega}\left(\frac{1}{C_1}+\frac{1}{C_2}\right)\mp 2\mathrm{i}\omega M}. \tag{20}$$

当只有电阻元件, 则得两个电阻并联的公式为$R=R_1R_2/(R_1+R_2)$; 当只有电容元件, 则得两个电容并联的公式$C=C_1+C_2$; 当只有电感元件, 则得两个电感并联的公式$L=L_1L_2/(L_1+L_2)$; 当只有互感元件, 则得两个互感并联的公式$L=(L_1L_2-M^2)/(L_1+L_2\mp M)$.

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Deriving A Formula from Field Theory for Four Kinds of Series-Parallel Elements with Mutual Induction

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Abstract By applying the law of energy transformation and conservation in electromagnetism to a sinusoidal steady-state linear exchange network, the aauthors obtain a new expression of the law in which the merits of such energy forms as resistance, capacitance, self-induction and mutual induction of four kinds of elements are embodied. By using these merits in this new expression and such concepts of network modern field theory as rate of complex impedance of the element and complex impedance in integral form, the specific complex impedances in integral form of four kinds of elements are given respectively. By linking the relationship of complex, voltage and current with the series-parallel connection of circuit, a formula of complex impedance is derived for four kinds of series and then series-parallel elements. The correctness of the formula is verified.

Keywords mutual induction, element of circuit, formula of series-parallel connection