

各向异性磁介质中载流矩形线圈空间磁场*

张焕明 林建兴 刘强生

(华侨大学电子工程系, 泉州 362011)

摘要 在给出的各向异性的毕奥-萨伐尔定律的基础上, 求出载流矩形线圈在各向异性磁介质中的空间磁场, 为研究磁各向异性的磁场提供了实例.

关键词 磁介质, 各向异性, 磁场, 载流矩形线圈

分类号 TM 154.3

本文求出载流矩形线圈在各向异性磁介质中的空间任意一点的磁场, 补充了文[1~7]给出的由一定电流分布所产生的磁各向异性磁场. 这在实际上, 为分析变压器以及电机中的矩形线圈在其周围磁各向异性介质中产生的磁效应, 提供了理论依据.

1 磁各向异性介质中毕奥-萨伐尔定律

当磁介质为线性各向异性时, 它的电磁性质方程为 $B_i = \sum_{j=1}^3 \mu_{ij} H_j (i=1, 2, 3)$. 当磁导率张量 μ_{ij} 的三个主轴与直角坐标轴 x, y, z 重合时和当电流作线分布时, 文[3]给出磁各向异性的毕奥-萨伐尔定律为

$$B(x) = \frac{\mu}{4\pi} \cdot \int \frac{Idl \times R}{\left(\frac{R_1^2}{\mu_{11}} + \frac{R_2^2}{\mu_{22}} + \frac{R_3^2}{\mu_{33}}\right)^{3/2}}, \quad (1)$$

式中

$$\mu = \sqrt{\frac{\mu_{11}}{\mu_{22}\mu_{33}}} \mathbf{i}\mathbf{i} + \sqrt{\frac{\mu_{22}}{\mu_{11}\mu_{33}}} \mathbf{j}\mathbf{j} + \sqrt{\frac{\mu_{33}}{\mu_{11}\mu_{22}}} \mathbf{k}\mathbf{k}, \quad (2)$$

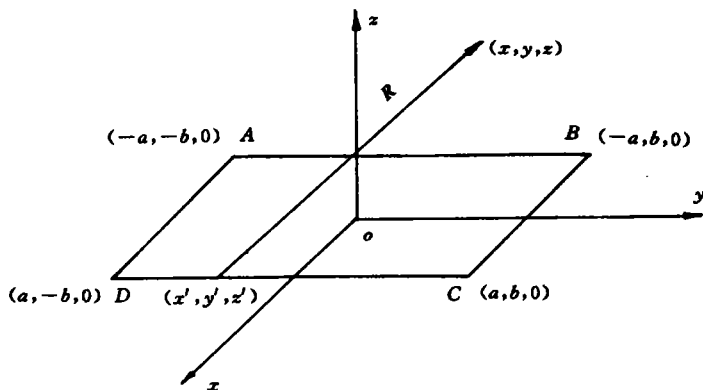
$$\begin{aligned} R &= R_1 \mathbf{i} + R_2 \mathbf{j} + R_3 \mathbf{k} \\ &= (x - x') \mathbf{i} + (y - y') \mathbf{j} + (z - z') \mathbf{k}, \end{aligned} \quad (3)$$

(x, y, z) 是场点坐标, (x', y', z') 是源点坐标.

2 载流矩形线圈在各向异性介质中的空间磁场

设有一载流为 I , 边长为 $2a$ 和 $2b$ 的矩形线圈, 置于 xoy 平面内, 而且其中心与坐标原点重合, 如附图所示. 依式(1)此线圈关于位于空间中任意一点 (x, y, z) 的磁场为

* 本文 1996-01-19 收到; 福建省自然科学基金资助项目

附图 位于 xy 面内载流为 I 的矩形线圈

$$\begin{aligned}
 B &= \frac{\mu}{4\pi} \cdot \int \frac{Idl \times R}{\left(\frac{R_1^2}{\mu_{11}} + \frac{R_2^2}{\mu_{22}} + \frac{R_3^2}{\mu_{33}}\right)^{3/2}} \\
 &= I \frac{\mu}{4\pi} \cdot \int_{-b}^b \frac{dy' j \times [(x+a)i + (y-y')j + zk]}{\left[\frac{(x+a)^2}{\mu_{11}} + \frac{(y-y')^2}{\mu_{22}} + \frac{z^2}{\mu_{33}}\right]^{3/2}} \\
 &\quad + I \frac{\mu}{4\pi} \cdot \int_{-a}^a \frac{dx' j \times [(x-x')i + (y-b)j + zk]}{\left[\frac{(x-x')^2}{\mu_{11}} + \frac{(y-b)^2}{\mu_{22}} + \frac{z^2}{\mu_{33}}\right]^{3/2}} \\
 &\quad + I \frac{\mu}{4\pi} \cdot \int_b^{-b} \frac{dy' (-j) \times [(x+a)i + (y-y')j + zk]}{\left[\frac{(x-a)^2}{\mu_{11}} + \frac{(y-y')^2}{\mu_{22}} + \frac{z^2}{\mu_{33}}\right]^{3/2}} \\
 &\quad + I \frac{\mu}{4\pi} \cdot \int_a^{-a} \frac{dx' (-j) \times [(x-x')i + (y-b)j + zk]}{\left[\frac{(x-x')^2}{\mu_{11}} + \frac{(y+b)^2}{\mu_{22}} + \frac{z^2}{\mu_{33}}\right]^{3/2}} \\
 &= I \frac{\mu}{4\pi} \cdot \left\{ [zi - (x+a)k] \int_{-b}^b \frac{dy'}{\left[\frac{(x+a)^2}{\mu_{11}} + \frac{(y-y')^2}{\mu_{22}} + \frac{z^2}{\mu_{33}}\right]^{3/2}} \right. \\
 &\quad + [(y-b)k - zj] \int_{-a}^a \frac{dx'}{\left[\frac{(x-x')^2}{\mu_{11}} + \frac{(y-b)^2}{\mu_{22}} + \frac{z^2}{\mu_{33}}\right]^{3/2}} \\
 &\quad + [(x-a)k - zi] \int_b^{-b} \frac{dy'}{\left[\frac{(x-a)^2}{\mu_{11}} + \frac{(y-y')^2}{\mu_{22}} + \frac{z^2}{\mu_{33}}\right]^{3/2}} \\
 &\quad \left. + [zj - (y+b)k] \int_a^{-a} \frac{dx'}{\left[\frac{(x-x')^2}{\mu_{11}} + \frac{(y+b)^2}{\mu_{22}} + \frac{z^2}{\mu_{33}}\right]^{3/2}} \right\}. \quad (4)
 \end{aligned}$$

分别用 c, d, e, f 代表上式中右边四项的积分, 经积分之后, 它们各为

$$c = \frac{b-y}{\left[\frac{(x+a)^2}{\mu_{11}} + \frac{z^2}{\mu_{33}}\right] \left[\frac{(x+a)^2}{\mu_{11}} + \frac{(b-y)^2}{\mu_{22}} + \frac{z^2}{\mu_{33}}\right]^{\frac{1}{2}}}$$

$$\begin{aligned}
 & + \frac{b+y}{\left[\frac{(x+a)^2}{\mu_{11}} + \frac{z^2}{\mu_{33}} \right] \left[\frac{(x+a)^2}{\mu_{11}} + \frac{(b+y)^2}{\mu_{22}} + \frac{z^2}{\mu_{33}} \right]^{\frac{1}{2}}}, \\
 e = & \frac{y-b}{\left[\frac{(x-a)^2}{\mu_{11}} + \frac{z^2}{\mu_{33}} \right] \left[\frac{(x-a)^2}{\mu_{11}} + \frac{(y-b)^2}{\mu_{22}} + \frac{z^2}{\mu_{33}} \right]^{\frac{1}{2}}} \\
 & - \frac{y+b}{\left[\frac{(x-a)^2}{\mu_{11}} + \frac{z^2}{\mu_{33}} \right] \left[\frac{(x-a)^2}{\mu_{11}} + \frac{(y+b)^2}{\mu_{22}} + \frac{z^2}{\mu_{33}} \right]^{\frac{1}{2}}}, \\
 d = & \frac{a-x}{\left[\frac{(y-b)^2}{\mu_{22}} + \frac{z^2}{\mu_{33}} \right] \left[\frac{(a-x)^2}{\mu_{11}} + \frac{(y-b)^2}{\mu_{22}} + \frac{z^2}{\mu_{33}} \right]^{\frac{1}{2}}} \\
 & + \frac{a+x}{\left[\frac{(y-b)^2}{\mu_{22}} + \frac{z^2}{\mu_{33}} \right] \left[\frac{(a+x)^2}{\mu_{11}} + \frac{(y-b)^2}{\mu_{22}} + \frac{z^2}{\mu_{33}} \right]^{\frac{1}{2}}}, \\
 f = & \frac{x-a}{\left[\frac{(y+b)^2}{\mu_{22}} + \frac{z^2}{\mu_{33}} \right] \left[\frac{(x-a)^2}{\mu_{11}} + \frac{(y+b)^2}{\mu_{22}} + \frac{z^2}{\mu_{33}} \right]^{\frac{1}{2}}} \\
 & - \frac{x+a}{\left[\frac{(y+b)^2}{\mu_{22}} + \frac{z^2}{\mu_{33}} \right] \left[\frac{(x+a)^2}{\mu_{11}} + \frac{(y+b)^2}{\mu_{22}} + \frac{z^2}{\mu_{33}} \right]^{\frac{1}{2}}}.
 \end{aligned}$$

代入式(4)并利用式(2),结果为

$$\begin{aligned}
 B = & i \frac{Iz}{4\pi} \sqrt{\frac{\mu_{11}}{\mu_{22}\mu_{33}}} \left\{ \frac{1}{\frac{(x+a)^2}{\mu_{11}} + \frac{z^2}{\mu_{33}}} \left[\frac{b-y}{\sqrt{\frac{(x+a)^2}{\mu_{11}} + \frac{(b-y)^2}{\mu_{22}} + \frac{z^2}{\mu_{33}}}} \right. \right. \\
 & \left. \left. + \frac{b+y}{\sqrt{\frac{(x+a)^2}{\mu_{11}} + \frac{(b+y)^2}{\mu_{22}} + \frac{z^2}{\mu_{33}}}} \right] - \frac{1}{\frac{(x-a)^2}{\mu_{11}} + \frac{z^2}{\mu_{33}}} \right. \\
 & \left. \times \left[\frac{b-y}{\sqrt{\frac{(x-a)^2}{\mu_{11}} + \frac{(b-y)^2}{\mu_{22}} + \frac{z^2}{\mu_{33}}}} + \frac{b+y}{\sqrt{\frac{(x-a)^2}{\mu_{11}} + \frac{(b+y)^2}{\mu_{22}} + \frac{z^2}{\mu_{33}}}} \right] \right\} \\
 & + j \frac{Iz}{4\pi} \sqrt{\frac{\mu_{22}}{\mu_{11}\mu_{33}}} \left\{ \frac{1}{\frac{(y+b)^2}{\mu_{22}} + \frac{z^2}{\mu_{33}}} \left[\frac{a-x}{\sqrt{\frac{(a-x)^2}{\mu_{11}} + \frac{(y+b)^2}{\mu_{22}} + \frac{z^2}{\mu_{33}}}} \right. \right. \\
 & \left. \left. + \frac{a+x}{\sqrt{\frac{(a+x)^2}{\mu_{11}} + \frac{(y+b)^2}{\mu_{22}} + \frac{z^2}{\mu_{33}}}} \right] - \frac{1}{\frac{(y-b)^2}{\mu_{22}} + \frac{z^2}{\mu_{33}}} \right. \\
 & \left. \times \left[\frac{a-x}{\sqrt{\frac{(y-b)^2}{\mu_{22}} + \frac{(a-x)^2}{\mu_{11}} + \frac{z^2}{\mu_{33}}}} + \frac{a+x}{\sqrt{\frac{(y-b)^2}{\mu_{22}} + \frac{(a+x)^2}{\mu_{11}} + \frac{z^2}{\mu_{33}}}} \right] \right\}
 \end{aligned}$$

$$\begin{aligned}
& \times \left[\frac{a-x}{\sqrt{\frac{(a-x)^2}{\mu_{11}} + \frac{(y-b)^2}{\mu_{22}} + \frac{z^2}{\mu_{33}}}} + \frac{a+x}{\sqrt{\frac{(a+x)^2}{\mu_{11}} + \frac{(y+b)^2}{\mu_{22}} + \frac{z^2}{\mu_{33}}}} \right] \Bigg\} \\
& + k \frac{I}{4\pi} \sqrt{\frac{\mu_{33}}{\mu_{11}\mu_{22}}} \left\{ \frac{x-a}{\sqrt{\frac{(x-a)^2}{\mu_{11}} + \frac{z^2}{\mu_{33}}}} \left[\frac{b-y}{\sqrt{\frac{(x-a)^2}{\mu_{11}} + \frac{(b-y)^2}{\mu_{22}} + \frac{z^2}{\mu_{33}}}} \right. \right. \\
& \left. \left. + \frac{b+y}{\sqrt{\frac{(x-a)^2}{\mu_{11}} + \frac{(b+y)^2}{\mu_{22}} + \frac{z^2}{\mu_{33}}}} \right] + \frac{y-b}{\sqrt{\frac{(y-b)^2}{\mu_{22}} + \frac{z^2}{\mu_{33}}}} \right. \\
& \times \left[\frac{a-x}{\sqrt{\frac{(a-x)^2}{\mu_{11}} + \frac{(y-b)^2}{\mu_{22}} + \frac{z^2}{\mu_{33}}}} + \frac{a+x}{\sqrt{\frac{(a+x)^2}{\mu_{11}} + \frac{(y-b)^2}{\mu_{22}} + \frac{z^2}{\mu_{33}}}} \right] \\
& - \frac{x+a}{\sqrt{\frac{(x+a)^2}{\mu_{11}} + \frac{z^2}{\mu_{33}}}} \left[\frac{b-y}{\sqrt{\frac{(x+a)^2}{\mu_{11}} + \frac{(b-y)^2}{\mu_{22}} + \frac{z^2}{\mu_{33}}}} \right. \\
& \left. \left. + \frac{b+y}{\sqrt{\frac{(x+a)^2}{\mu_{11}} + \frac{(b+y)^2}{\mu_{22}} + \frac{z^2}{\mu_{33}}}} \right] - \frac{y+b}{\sqrt{\frac{(y+b)^2}{\mu_{11}} + \frac{z^2}{\mu_{33}}}} \right. \\
& \times \left[\frac{a-x}{\sqrt{\frac{(a-x)^2}{\mu_{11}} + \frac{(y+b)^2}{\mu_{22}} + \frac{z^2}{\mu_{33}}}} + \frac{a+x}{\sqrt{\frac{(a+x)^2}{\mu_{11}} + \frac{(y+b)^2}{\mu_{22}} + \frac{z^2}{\mu_{33}}}} \right] \Bigg\}. \quad (5)
\end{aligned}$$

3 讨论

(1) 当场点位于 z 轴上时

$$B = -\frac{kI}{\pi} \sqrt{\frac{\mu_{33}}{\mu_{11}\mu_{22}}} \frac{ab}{\left(\frac{a^2}{\mu_{11}} + \frac{b^2}{\mu_{22}} + \frac{z^2}{\mu_{33}}\right)^{\frac{1}{2}}} \left(\frac{1}{\frac{a^2}{\mu_{11}} + \frac{z^2}{\mu_{33}}} + \frac{1}{\frac{b^2}{\mu_{22}} + \frac{z^2}{\mu_{33}}} \right);$$

(2) 当场点位于 y 轴上时

$$\begin{aligned}
B = & -k \frac{I}{2\pi} \sqrt{\frac{\mu_{33}}{\mu_{11}\mu_{22}}} \left[\frac{\mu_{11}}{a} \left[\frac{b-y}{\left[\frac{a^2}{\mu_{11}} + \frac{(b-y)^2}{\mu_{22}}\right]^{\frac{1}{2}}} + \frac{b+y}{\left[\frac{a^2}{\mu_{11}} + \frac{(b+y)^2}{\mu_{22}}\right]^{\frac{1}{2}}} \right] \right. \\
& \left. + \frac{\mu_{22}}{b-y} \frac{a}{\left[\frac{a^2}{\mu_{11}} + \frac{(y-b)^2}{\mu_{22}}\right]^{\frac{1}{2}}} + \frac{\mu_{22}}{y+b} \frac{a}{\left[\frac{a^2}{\mu_{11}} + \frac{(y+b)^2}{\mu_{22}}\right]^{\frac{1}{2}}} \right];
\end{aligned}$$

(3) 当场点位于 x 轴上时

$$B = -k \frac{I}{2\pi} \sqrt{\frac{\mu_{33}}{\mu_{11}\mu_{22}}} \left[\frac{\mu_{22}}{b} \left[\frac{a-x}{\left[\frac{(a-x)^2}{\mu_{11}} + \frac{b^2}{\mu_{22}}\right]^{\frac{1}{2}}} + \frac{a+x}{\left[\frac{(a+x)^2}{\mu_{11}} + \frac{b^2}{\mu_{22}}\right]^{\frac{1}{2}}} \right] \right]$$

$$+ \frac{\mu_{11}}{a-x} \frac{b}{\left[\frac{(a-x)^2}{\mu_{11}} + \frac{b^2}{\mu_{22}} \right]^{\frac{1}{2}}} + \frac{\mu_{11}}{a+x} \frac{b}{\left[\frac{(a+x)^2}{\mu_{11}} + \frac{b^2}{\mu_{22}} \right]^{\frac{1}{2}}};$$

(4) 当场点位于原点时

$$B = -\frac{kI}{\pi} \left(\frac{b\mu_{11}}{a} + \frac{a\mu_{22}}{b} \right) \cdot \left(\sqrt{\frac{a^2}{\mu_{11}} + \frac{b^2}{\mu_{22}}} \right)^{-1} \cdot \sqrt{\frac{\mu_{33}}{\mu_{11}\mu_{22}}},$$

若 $\mu_{11} = \mu_{22} = \mu_{33} = \mu$ 时, $B = -(kI/\pi)\mu \cdot \sqrt{a^2 + b^2}/ab$.

由以上讨论可知,介质的各向异性对磁场分布的影响,与原点等距离但分别位于 x 轴和 y 轴上的两点.磁感应强度大小不相同,显示出了各向异性;也使位于 z 轴上的磁场有赖于线圈的轴线,相对于各向同性介质中三个主轴的方位.在变压器或电机中,若要轴线上有最大磁场,就要选取 $\epsilon_{11}, \epsilon_{22}, \epsilon_{33}$ 中最大的一个主轴为线圈轴线.最后,介质为各向同性特殊的情况,验证了式(5)的正确性.

本文承陈荣年教授和苏武浔老师的指导,特致谢意.

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Spatial Magnetic Field of Rectangular Charge-Carried Coil in Anisotropic Magnetic Medium

Zhang Huanming Lin Jianxing Liu Qiangsheng

(Dept. of Electron. Eng., Huaqiao Univ., 362011, Quanzhou)

Abstract In the light of anisotropic biot-savart law that has been given, the authors solve the spatial magnetic field of rectangular charge-carried coil in anisotropic magnetic medium. Thus an example is offered for studying anisotropic magnetic field.

Keywords magnetic medium, anisotropy, magnetic field, rectangular charge-carried coil