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一族绝对稳定的高精度差分格式*

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摘要 建立了解三维抛物型方程的一族含参数的绝对稳定的高精度差分格式. 李荣华等的结果可以看作两层差分格式的特例. 进而,在特殊情况($\theta=0, r=1/6$)下,我们得到显式差分格式. 我们证明了这些格式对任意选取的参数 $\theta\leqslant 1/3$ 都是绝对稳定的且其截断误差阶为 $0(\Delta t)^2+\Delta t(\Delta x)^2+(\Delta x)^4)=0((\Delta t)^2)$.

关键词 差分格式,抛物型方程,三维,高准确度,绝对稳定

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渗流扩散、热传导等很多领域经常会遇到求解如下的三维抛物型方程的初边值问题

$$\left.\begin{array}{ll}\dfrac{\partial u}{\partial t}=\dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial^2 u}{\partial y^2}+\dfrac{\partial^2 u}{\partial z^2} & (0\leqslant x,y,z\leqslant 1, t>0),\\ u(0,y,z,t)=f_1(y,z,t),\ u(1,y,z,t)=f_2(y,z,t) & (0\leqslant y,z\leqslant 1,t>0),\\ u(x,0,z,t)=g_1(x,z,t),\ u(x,1,z,t)=g_2(x,z,t) & (0\leqslant x,z\leqslant 1,t>0),\\ u(x,y,0,t)=h_1(x,y,t),\ u(x,y,1,t)=h_2(x,y,t) & (0\leqslant x,y\leqslant 1,t>0),\\ u(x,y,z,o)=\varphi(x,y,z) & (0\leqslant x,y,z\leqslant 1).\end{array}\right\}\quad(1)$$

解上述问题的古典显式或隐式格式或 Crank-Nicholson 格式的精度均不高,截断误差仅为 $0(\Delta t+(\Delta x)^2)$或 $0((\Delta t)^2+(\Delta x)^2)$. 在文〔1〕中构造了精度较高且绝对稳定的差分格式,其截断误差达到 $0(\Delta t)^2+(\Delta x)^4+\dfrac{(\Delta x)^6}{\Delta t})$,但却是三层的隐式格式,徒增了很多计算量与存储量. 本文构造了一族两层含参数绝对稳定、高精度的隐式差分格式(特殊情况下是显式),包含了文〔2〕中的高精度恒稳格式. 当参数 $\theta=0$,网格化 $r=1/6$ 时是一个两层的显式差分格式. 可以证明,本文构造的一族含参数高精度格式当 $\theta\leqslant 1/3$ 是恒稳的,且其截断误差阶数为 $0(\Delta t)^2+\Delta t(\Delta x)^2+(\Delta x)^4)$.

1 高精度差分格式的构造

设 Δt 为时间步长,$\Delta x,\Delta y,\Delta z$ 分别为 x,y,z 方向的空间步长. 为简便计,设 $\Delta x=\Delta y=\Delta z=1/N$($N$ 为正整数),用如下的含参数的差分方程逼近微分方程(1),有

$$\frac{\omega_{i,j,k}^{n+1}-\omega_{i,j,k}^{n}}{\Delta t}=\frac{1}{(\Delta x)^2}\left\{\frac{\theta_1}{4}\square+\frac{\theta_2}{2}\Diamond\right\}\omega_{i,j,k}^{n+1}+\frac{1}{(\Delta x)^2}\left\{\frac{\theta_3}{4}\square+\frac{\theta_4}{2}\Diamond\right\}\omega_{i,j,k}^{n},\qquad(2)$$

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其中 $\omega_{i,j,k}^n$ 表示在节点$(i\Delta x, j\Delta y, k\Delta z, n\Delta t)$处的网格函数值,且记

$$\left.\begin{aligned}
&\square\omega_{i,j,k}^n = ({}_x\square + {}_y\square + {}_z\square)\omega_{i,j,k}^n,\\
&{}_x\square\omega_{i,j,k}^n = \omega_{i+1,j+1,k}^n + \omega_{i-1,j+1,k}^n + \omega_{i+1,j-1,k}^n + \omega_{i-1,j-1,k}^n - 4\omega_{i,j,k}^n,\\
&\Diamond\omega_{i,j,k}^n = ({}_x\Diamond + {}_y\Diamond + {}_z\Diamond)\omega_{i,j,k}^n,\\
&{}_x\Diamond\omega_{i,j,k}^n = \omega_{i+1,j,k}^n + \omega_{i,j+1,k}^n + \omega_{i-1,j,k}^n + \omega_{i,j-1,k}^n - 4\omega_{i,j,k}^n,
\end{aligned}\right\}\tag{3}$$

其余类推.$\theta_1,\theta_2,\theta_3,\theta_4$ 为待定参数,适当选择这些参数,可以使差分格式(2)逼近微分方程(1)具有尽可能高阶的离散误差、而且有较好的稳定性.今后为简便计,略去下标 i,j,k 而记 $\omega_{i,j,k}^n=\omega^n$,其余类推.当微分方程(1)的解充分光滑时,有如下关系式成立

$$\frac{\partial^n}{\partial t^n}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)^m\omega = \left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)^{m+2n}\omega \quad (m,n\text{ 为非负整数}). \tag{4}$$

在节点 $(i\Delta x,\ j\Delta y,\ k\Delta z,\ (n+\frac{1}{2})\Delta t$ 处进行 Tayhor 展开并利用关系式(4)可得

$$\begin{aligned}
&\frac{\omega^{n+1}-\omega^n}{\Delta t} - \frac{1}{(\Delta x)^2}\left\{\frac{\theta_1}{4}\square + \frac{\theta_2}{2}\Diamond\right\}\omega^{n+1} - \frac{1}{(\Delta x)^2}\left\{\frac{\theta_3}{4}\square + \frac{\theta_4}{2}\Diamond\right\}\omega^n\\
&=(1-\theta_1-\theta_2-\theta_3-\theta_4)\frac{\partial\omega^{n+1/2}}{\partial t} - \frac{\Delta t}{2}(\theta_1+\theta_2-\theta_3-\theta_4)\frac{\partial^2\omega^{n+1/2}}{\partial t^2}\\
&\quad-\frac{(\Delta x)^2}{12}(\theta_1+\theta_2+\theta_3+\theta_4)\frac{\partial^2\omega^{n+1/2}}{\partial t^2} - \frac{(\Delta x)^2}{12}[\theta_1+\theta_3-2(\theta_2+\theta_4)]\\
&\quad\times\left[\frac{\partial^4}{\partial x^2\partial y^2}+\frac{\partial^4}{\partial y^2\partial z^2}+\frac{\partial^4}{\partial z^2\partial x^2}\right]\omega^{n+1/2} - \frac{\Delta t(\Delta x)^2}{24}(\theta_1+\theta_2-\theta_3-\theta_4)\frac{\partial^3\omega^{n+1/2}}{\partial t^3}\\
&\quad-\frac{\Delta t(\Delta x)^2}{24}(\theta_1-\theta_3-2\theta_2+2\theta_4)\left(\frac{\partial^4}{\partial x^2\partial y^2}+\frac{\partial^4}{\partial y^2\partial z^2}+\frac{\partial^4}{\partial z^2\partial x^2}\right)\\
&\quad+0((\Delta t)^2+(\Delta x)^4).
\end{aligned}\tag{5}$$

于是,我们得到差分格式(2)的截断误差阶如下:

(Ⅰ)当满足下列相容性条件时截断误差阶为 $0(\Delta t+(\Delta x)^2)$,

$$\theta_1+\theta_2+\theta_3+\theta_4=1\ ; \tag{6}$$

(Ⅱ)当满足下列条件时截断误差阶为 $0((\Delta t)^2+(\Delta x)^2)$,

$$\theta_1+\theta_2=\theta_3+\theta_4=\frac{1}{2}\ ; \tag{7}$$

(Ⅲ)当满足下列条件时截断误差阶为 $0((\Delta t)^2+\Delta t(\Delta x)^2+(\Delta x)^4)$,

$$\theta_2+\theta_4=\frac{1}{3},\quad \theta_1+\theta_2=\frac{1}{2}-\frac{1}{12r},\quad \theta_1+\theta_3=\frac{2}{3}. \tag{8}$$

丢掉截断误差后,便得形如式(2)的差分格式.下面指出一些熟知的差分格式及本文所得的新格式.

(1) 截断误差为 $0(\Delta t+(\Delta x)^2)$的格式,即参数满足相容性条件的差分格式(2),特别地有

(a) $\theta_1=\theta_2=\theta_3=0,\theta_4=1$,即古典显式格式

$$\frac{\omega^{n+1}-\omega^n}{\Delta t}=\frac{1}{2(\Delta x)^2}\Diamond\omega^n, \tag{9}$$

(b) $\theta_1=\theta_3=\theta_4=0,\theta_2=1$,即古典隐式格式

$$\frac{\omega^{n+1}-\omega^n}{\Delta t}=\frac{1}{2(\Delta x)^2}\Diamond\omega^{n+1}, \tag{10}$$

(c) $\theta_1=1,\theta_2=\theta_3=\theta_4=0$，即新格式

$$\frac{\omega^{n+1}-\omega^n}{\Delta t}=\frac{1}{4(\Delta x)^2}\Box\omega^{n+1},\tag{11}$$

(d) $\theta_1=\frac{1}{3},\theta_2=\frac{2}{3},\theta_3=\theta_4=0$，即新格式

$$\frac{\omega^{n+1}-\omega^n}{\Delta t}=\{\frac{1}{12}\Box\omega^{n+1}+\frac{1}{3}\Diamond\omega^n\}\frac{1}{(\Delta x)^2},\tag{12}$$

此类格式可列出很多，从略．

(2) 截断误差为 $0((\Delta t)^2+(\Delta x)^2)$ 的差分格式．若令 $\theta_1=\theta,\theta_3=\eta$，则由条件(7)得

$$\theta_1=\theta,\theta_2=\frac{1}{2}-\theta,\qquad \theta_3=\eta,\theta_4=\frac{1}{2}-\eta,\tag{13}$$

于是得截断误差为 $0((\Delta t)^2+(\Delta x)^2)$ 的一般格式为

$$\frac{\omega^{n+1}-\omega^n}{\Delta t}=\frac{1}{4}\{\theta\Box+(1-2\theta)\Diamond\}\frac{\omega^{n+1}}{(\Delta x)^2}+\frac{1}{4}\{\eta\Box+(1-2\eta)\Diamond\}\frac{\omega^n}{(\Delta x)^2},\tag{14}$$

特别地，有下列 (a)～(c)等等．其中

(a) $\theta_1=\theta_3=\frac{1}{2},\theta_2=\theta_4=0$，即新格式

$$\frac{\omega^{n+1}-\omega^n}{\Delta t}=\frac{1}{8(\Delta x)^2}\Box\omega^{n+1}+\frac{1}{8(\Delta x)^2}\Box\omega^n,\tag{15}$$

(b) $\theta_1=\theta_3=\frac{1}{6},\theta_2=\theta_4=\frac{1}{3}$，即新格式

$$\frac{\omega^{n+1}-\omega^n}{\Delta t}=\frac{1}{24(\Delta x)^2}\{(\Box+4\Diamond)\omega^{n+1}+(\Box+4\Diamond)\omega^n\},\tag{16}$$

(c) $\theta_1=\theta_2=\theta_3=\theta_4=\frac{1}{4}$，即新格式

$$\frac{\omega^{n+1}-\omega^n}{\Delta t}=\frac{1}{16(\Delta x)^2}\{(\Box+2\Diamond)\omega^{n+1}+(\Box+2\Diamond)\omega^n\}.\tag{17}$$

(3) 截断误差为 $0((\Delta t)^2+(\Delta t)(\Delta x)^2+(\Delta x)^4=0((\Delta t)^2)$ 的差分格式．若令 $\theta_1=\theta$，则由条件(8)得

$$\theta_1=\theta,\qquad \theta_2=\frac{1}{2}-\frac{1}{12r}-\theta,$$

$$\theta_3=\frac{2}{3}-\theta,\qquad \theta_4=-\frac{1}{6}+\frac{1}{12r}+\theta.\tag{18}$$

于是截断误差为 $0(\Delta t)^2$的一般格式为

$$\frac{\omega^{n+1}-\omega^n}{\Delta t}=\frac{1}{(\Delta x)^2}\{[\frac{\theta}{4}\Box+(\frac{1}{4}-\frac{1}{24r}-\frac{\theta}{2})\Diamond]\omega^{n+1}$$

$$+[(\frac{1}{6}-\frac{\theta}{4})\Box+(-\frac{1}{12}+\frac{1}{24r}+\frac{\theta}{2})\Diamond\omega^n\}.\tag{19a}$$

注意到

$$_z\Diamond\omega^n=(\delta_x^2+\delta_y^2)\omega^n,\quad _z\Box\omega^n=2(\delta_x^2+\delta_y^2)\omega^n+\delta_x^2\delta_y^2\omega^n.\tag{20}$$

则有

$$\Diamond\omega^n=2(\delta_x^2+\delta_y^2+\delta_z^2)\omega^n,\quad \Box\omega^n=4(\delta_x^2+\delta_y^2+\delta_z^2)\omega^n+(\delta_x^2\delta_y^2+\delta_z^2\delta_x^2)\omega^n,\tag{21}$$

其中 $\delta_x^2,\delta_y^2,\delta_z^2$分别为 x,y,z 方向的二阶中心差分．则一般格式(19a)可改写为

$$\frac{\omega^{n+1}-\omega^n}{\Delta t}=\frac{1}{(\Delta x)^2}\{(\frac{1}{2}-\frac{1}{12r})(\delta_x^2+\delta_y^2+\delta_z^2)+\frac{\theta}{4}(\delta_x^2\delta_y^2+\delta_y^2\delta_z^2+\delta_z^2\delta_x^2)\}\omega^{n+1}$$
$$+\frac{1}{(\Delta x)^2}\{(\frac{1}{2}+\frac{1}{12r})(\delta_x^2+\delta_y^2+\delta_z^2)$$
$$+(\frac{1}{6}-\frac{\theta}{4})(\delta_x^2\delta_y^2+\delta_y^2\delta_z^2+\delta_z^2\delta_x^2)\}\omega^n, \tag{19b}$$

特别地有

(a) $\theta=\theta_1=\theta_3=\frac{1}{3},\theta_2=\frac{1}{6}-\frac{1}{12r},\theta_4=\frac{1}{6}+\frac{1}{12r}$，得

$$\frac{\omega^{n+1}-\omega^n}{\Delta t}=\frac{1}{(\Delta x)^2}\{\frac{1}{12}\Box+(\frac{1}{12}-\frac{1}{24r})\Diamond\}\omega^{n+1}$$
$$+\frac{1}{(\Delta x)^2}\{\frac{1}{12}\Box+(\frac{1}{12}+\frac{1}{24r})\Diamond\}\omega^n. \tag{22}$$

(b) $\theta=\theta_1=\frac{1}{3}-\frac{1}{12r},\theta_2=\theta_4=\frac{1}{6},\theta_3=\frac{1}{3}+\frac{1}{12r}$,即为新格式

$$\frac{\omega^{n+1}-\omega^n}{\Delta t}=\frac{1}{(\Delta x)^2}\{[(\frac{1}{12}-\frac{1}{48r})\Box+\frac{1}{12}\Diamond]\omega^{n+1}$$
$$+[(\frac{1}{12}+\frac{1}{48r})\Box+\frac{1}{12}\Diamond]\omega^n. \tag{23}$$

(c) $\theta=\theta_1=0,\theta_2=\frac{1}{2}-\frac{1}{12r},\theta_3=\frac{2}{3},\theta_4=\frac{1}{6}+\frac{1}{12r}$，即得文[2]格式

$$\frac{\omega^{n+1}-\omega^n}{\Delta t}=\frac{1}{(\Delta x)^2}(\frac{1}{4}-\frac{1}{24r}\Diamond\omega^{n+1}+\frac{1}{(\Delta x)^2}\{\frac{1}{6}\Box+(-\frac{1}{12}+\frac{1}{24r})\Diamond\}\omega^n, \tag{24a}$$

即

$$\frac{\omega^{n+1}-\omega^n}{\Delta t}=\frac{1}{(\Delta x)^2}\{(\frac{1}{2}-\frac{1}{12r})(\delta_x^2+\delta_y^2+\delta_z^2)\omega^{n+1}$$
$$+\frac{1}{(\Delta x)^2}(\frac{1}{2}+\frac{1}{12r})(\delta_x^2+\delta_y^2+\delta_z^2)\omega^n$$
$$+\frac{1}{6(\Delta x)^2}(\delta_x^2\delta_y^2+\delta_y^2\delta_z^2+\delta_z^2\delta_x^2)\}\omega^n. \tag{24b}$$

(d) $\theta=\theta_1=\theta_2=0,r=\frac{1}{6}$，即得两层高精度显式格式(此时 $\theta_3=\frac{2}{3},\theta_4=\frac{1}{3}$)

$$\omega^{n+1}=\{1+\frac{1}{36}\Box+\frac{1}{36}\Diamond\}\omega^n, \tag{25a}$$

即

$$\omega^{n+1}=\{1+\frac{1}{6}(\delta_x^2+\delta_y^2+\delta_z^2)+\frac{1}{36}(\delta_x^2\delta_y^2+\delta_y^2\delta_z^2+\delta_z^2\delta_x^2)\}\omega^n. \tag{25b}$$

2 差分格式的稳定性

定理 1 若参数 $\theta_1,\theta_2,\theta_3,\theta_4$满足下列条件之一：(i) $\theta_1+\theta_3\geqslant 0,\theta_2+\theta_4\geqslant 0$，且 $\theta_4\leqslant\theta_1,\theta_4\leqslant\theta_2$；(ii) $\theta_1+\theta_2=\theta_3+\theta_4=\frac{1}{2}$，且 $\theta_1+\theta_3\leqslant 1,\theta_1\leqslant\theta_3$；(iii) $(\theta_1+\theta_2+\theta_3+\theta_4)(S_1^2+S_2^2+S_3^2)-(\theta_1+\theta_3)(S_1^2S_2^2+S_2^2S_3^2+S_3^2S_1^2)\geqslant 0$，或 $2r\{[(\theta_3+\theta_4)-(\theta_1+\theta_2)](S_1^2+S_2^2+S_3^2)+2r(\theta_1-\theta_3)(S_1^2S_2^2+S_2^2S_3^2+S_3^2S_1^2)\}\leqslant 1$，则差分格式(2)无条件稳定.

证明 根据分离变量法，令

$$\omega_{i,j,k}^{n}=\omega^{n}\mathrm{e}^{(p\pi x_i+q\pi y_j+l\pi z_k)i^*}=\sqrt{-1},\tag{26}$$

记

$$\left.\begin{aligned}&A=S_1^2+S_2^2+S_3^2=\sin^2(\frac{p\pi}{2N})+\sin^2(\frac{q\pi}{2N})+\sin^2(\frac{l\pi}{2N}),\\&B=S_1^2+S_2^2+S_3^2-(S_1^2S_2^2+S_2^2S_3^2+S_3^2S_1^2),\\&S_1=\sin\frac{p\pi}{2N},\ S_2=\sin\frac{q\pi}{2N},\ S_3=\sin\frac{l\pi}{2N},\end{aligned}\right\}\tag{27}$$

于是

$$\Diamond\omega^n=-8A\omega^n,\quad \Box\omega^n=-16B\omega^n,\tag{28}$$

其中 $0\leqslant S_1,S_2,S_3\leqslant 1$，$0\leqslant A,B\leqslant 3$. 由式(26)～(28)可得差分格式(2)的传播矩阵为

$$\begin{aligned}G&=\frac{1-4r\theta_3B-4r\theta_4A}{1+4r\theta_1B+4r\theta_2A}\\&=\frac{1-4r(\theta_3+\theta_4)(S_1^2+S_2^2+S_3^2)+4r\theta_3(S_1^2S_2^2+S_2^2S_3^2+S_3^2S_1^2)}{1+4r(\theta_1+\theta_2)(S_1^2+S_2^2+S_3^2)-4r\theta_1(S_1^2S_2^2+S_2^2S_3^2+S_3^2S_1^2)}.\end{aligned}\tag{29}$$

若对任意的 r 和 $S_1^2,S_2^2,S_3^2\in[0,1]$ 均有 $|G|\leqslant 1$，则差分格式(2)无条件稳定．而

$$\begin{aligned}&|G|\leqslant 1\\&\Leftrightarrow -1-4r\theta_1B-4r\theta_2A\leqslant 1-4r\theta_3B-4r\theta_4A\leqslant 1+4r\theta_1B+4r\theta_2A\\&\Leftrightarrow\begin{cases}4r(\theta_1+\theta_3)B+4r(\theta_2+\theta_4)A\geqslant 0\\4r(\theta_3-\theta_1)B+4r(\theta_4-\theta_2)A\leqslant 2\end{cases}\\&\Leftrightarrow\begin{cases}(\theta_1+\theta_3)B+(\theta_2+\theta_4)A\geqslant 0\\r\{(\theta_3-\theta_1)B+(\theta_4-\theta_2)A\}\leqslant 1\end{cases}\\&\Leftrightarrow\begin{cases}(\theta_1+\theta_2+\theta_3+\theta_4)(S_1^2+S_2^2+S_3^2)\\\qquad-(\theta_1+\theta_3)(S_1^2S_2^2+S_2^2S_3^2+S_3^2S_1^2)\geqslant 0,\\2r(\theta_3+\theta_4-\theta_1-\theta_2)(S_1^2+S_2^2+S_3^2)\\\qquad+2r(\theta_1-\theta_3)(S_1^2S_2^2+S_2^2S_3^2+S_3^2S_1^2)\leqslant 1.\end{cases}\end{aligned}\tag{30}$$

由式(30)立得充分条件(i)；式(30)即充分条件(iii). 显然，当充分条件(ii)成立时式(30)也成立，从而完成了基本定理的证明．

推论 1 差分格式(10)，(11)，(12)均无条件稳定，而差分格式(9)的稳定性条件为 $r\leqslant\frac{1}{6}$.

证明 注意到差分格式(10)，(11)，(12)的 $\theta_1,\theta_2,\theta_4,\theta_4$ 取值，则由基本定理的充分条件 1 立得它们无条件稳定；而由充分条件(iii)立得格式(9)的稳定性条件为 $r\leqslant\frac{1}{6}$.

推论 2 差分格式(15)，(16)，(17)无条件稳定；而差分格式(14)的稳定性条件为 $\theta\leqslant\eta$，且 $\theta+\eta\leqslant 1$.

推论 3 高精度差分格式(19)当 $\theta\leqslant\frac{1}{3}$ 时无条件稳定．

证明 将式(18)代入基本定理的充分条件(iii)得其第一式为

$$(S_1^2+S_2^2+S_3^2)-\frac{2}{3}(S_1^2S_2^2+S_2^2S_3^2+S_3^2S_1^2)\geqslant 0,$$

自然成立;而其第二式成为

$$\frac{1}{3}(S_1^2+S_2^2+S_3^2)+2r(2\theta-\frac{2}{3})(S_1^2S_2^2+S_2^2S_3^2+S_3^2S_1^2)\leqslant 1\,.$$

可见,当 $\theta\leqslant\frac{1}{3}$ 时上式也成立,从而完成了推论3的证明.

根据推论3,注意到高精度格式(22)～(24)的 θ 取值均不超过1/3,于是立刻有

推论4 高精度差分格式(22)～(24)是无条件稳定的.

推论5 高精度两层显格式(25)(此时 $r=\frac{1}{6}$)是稳定的.

最后,注意到本文所构造的差分格式,当 $\theta_1+\theta_2+\theta_3+\theta_4=1$ 时是相容的,再由Lax的稳定性与收稳性的等价性定理可推得这些格式的收敛性.

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A Family of Absolutely Stable and High Accurate Difference Schemes

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Abstract For solving three-dimensional parabolic equation, the author establishes a family of absolutely stable and high accurate difference schemes with a parameter. The results of reference may be regarded as special cases of two-level difference scheme. Moreover, an explicit difference scheme is obtained under the special case of ($\theta=0$, $r=1/6$). The schemes are proved to be absolutely stable for $\theta\leqslant 1/3$, parameters chosen arbitrarily; and their truncation errors are in the order of $0((\Delta t)^2+\Delta t(\Delta x)^2+(\Delta t)^4=0(\Delta t)^2)$.

Keywords difference schemes, parabolic equations, three-dimension, high accuracy, absolutely stable