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含有小参数的守恒型方程的守恒型差分格式

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摘要　本文研究含有小参数的守恒型方程,证明相应的守恒型格式为一阶一致收敛.当边界条件退化为第一边值条件时,一致收敛性可进一步提高.

关键词　小参数,守恒型方程,守恒型格式,一致收敛

0 引言

Doolan, Miller, Schilder 在[1]中,研究守恒型方程

$$\begin{cases} \varepsilon(p(x)u'(x))' + (q(x)u(x))' + r(x)u(x) = f(x), \quad 0 < x < 1, \\ u(0) = A, \quad u(1) = B, \end{cases} \tag{1}$$

其中系数满足一定条件,证明非守恒型差分格式为一阶一致收敛.我们在文[2]考虑守恒型混合边值问题

$$\begin{cases} \varepsilon(p(x)u'(x))' + (q(x)u(x))' + r(x)u(x) = f(x), \quad 0 < x < 1, \\ \alpha^* u(0) - \beta^* u'(0) = A, \quad r^* u(1) + \delta^* u(1) = B, \end{cases} \tag{2}$$

其中系数满足一定条件,证明非守恒型差分格式也是一阶一致收敛.然而对于守恒型方程,人们更感兴趣于研究守恒型格式.文[3]作者讨论了方程

$$\begin{cases} -\varepsilon(p(x)u'(x))' + q(x)u(x) = f(x), \quad 0 < x < 1, \\ u(0) = A, u(1) = B, \end{cases} \tag{3}$$

的守恒型格式.

本文研究方程(2)的守恒型差分格式,用分离奇性法证明其一阶一致收敛.当边界条件为第一边值条件时,一致收敛性可进一步提高.

1 微分方程的性质

考虑方程

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$$
\begin{cases}
Lu(x) \equiv \varepsilon(p(x)u'(x))' + (q(x)u(x))' + r(x)u(x) = f(x), \quad 0 < x < 1, & \text{(4a)} \\
B_0 u(0) \equiv \alpha^* u(0) - \beta^* u'(0) = A, & \text{(4b)} \\
B_1 u(1) \equiv r^* u(1) + \delta^* u(1) = B, & \text{(4c)}
\end{cases}
$$

其中系数 $p(x),q(x),r(x),f(x)$ 在$[0,1]$上充分光滑且满足:$\bar{\alpha}>p(x)>\alpha>0,\bar{\beta}>q(x)>\beta>0,r(x)\leqslant 0,\bar{p}>p(x)\geqslant 0,q'(x)\leqslant 0$,系数 $\alpha^*,\beta^*,r^*,\delta^*\geqslant 0,\alpha^*+\beta^*>0,r^*+\delta^*>0,r^*+b^*>0$,这里 $b^*=\min(-r(x))=-\max r(x)$.根据文[2],有

引理1　方程(4)的解 $u(x)$ 满足 $u(x)=\bar{r}\cdot V(x)+z(x)$,其中

$$V(x)=\exp\left(-\frac{q(0)x}{p(0)\varepsilon}\right),$$

$$|z^{(i)}(x)|\leqslant c\{1+\varepsilon^{-i+1}\exp\left(-\frac{\alpha^* x}{\varepsilon}\right)\},\quad \frac{\beta}{\bar{\alpha}}>\alpha^*>0,$$

$$|\bar{r}|\leqslant\begin{cases} c, & \text{当 } \beta^*=0, \\ c\varepsilon, & \text{当 } \beta^*\neq 0. \end{cases}$$

2　守恒型差分格式

对区间$[0,1]$进行等距划分,$x_i=ih,i=0,1,\cdots,N,Nh=1$.考虑守恒型差分格式

$$L^h u_i \equiv \varepsilon\cdot\delta(\sigma_i(\rho)\cdot p(x_i)\delta u_i) + D_0(q(x_i)u_i) + r(x_i)u_i = f(x_i),\quad 1\leqslant i\leqslant N-1, \tag{5a}$$

$$B_0^h u_0 \equiv \alpha^* u_0 - \beta^*(u_1-u_0)/h = A, \tag{5b}$$

$$B_1^h u_N \equiv \gamma^* u_N + \delta^*(u_N-u_{N-1})/h = B, \tag{5c}$$

其中 $\sigma_i(\rho)=\frac{q(x_i-0.5h)\rho}{2p(x_i)}\coth\frac{q(x_i-0.5h)\rho}{2p(x_i)},\rho=h/\varepsilon$.下面极值原理成立.

引理2　若 $L^h u_i\leqslant 0,1\leqslant i\leqslant N-1,B_0^h u_0\geqslant 0,B_1^h u_N\geqslant 0$成立,则对$0\leqslant i\leqslant N$,恒有 $u_i\geqslant 0$.证明从略.

令 $R(x)=q(x_i-0.5h)/(2p(x)),S(x,\rho)=R(x)\rho\coth(R(x)\rho),x\in[\frac{h}{2},1-\frac{h}{2}]$,有引理3.

引理3　(1)$R(x)\geqslant\alpha^*$ 且 $R'(x)\leqslant 0$;(2)$\partial S(x,\rho)/\partial x\leqslant 0,\partial S(x,\rho)/\partial\rho\geqslant 0$;(3)$\varepsilon\cdot|S(x,\rho)-1|\leqslant c^* h$;(4)$\varepsilon\cdot S(x,\rho)\leqslant c$;(5)$\varepsilon|\partial S(x,\rho)/\partial x|\leqslant ch$;(6)$S(x,\rho)\geqslant 1$;(7)$Q(x_i,\rho)=-\frac{\varepsilon}{h}\left[S(x_i+\frac{1}{2})p(x_i+\frac{1}{2})-S(x_i-\frac{1}{2})P(x_i-\frac{1}{2})\right]+\frac{q_i-q_{i-1}}{2}\leqslant 0$;(8)当 $h\leqslant\varepsilon$ 时,$\varepsilon|\partial S(x,\rho)/\partial x|\leqslant ch^2/(h+\varepsilon)$.

证明　容易证明(1),(3)和(4)式成立.

(2)直接计算表明

$$\frac{\partial S(x,\rho)}{\partial x}=R'(x)\rho\frac{(1/2)\mathrm{sh}(2R(x)\rho)-R(x)\rho}{\mathrm{sh}^2(R(x)\rho)}\leqslant 0,$$

$$\frac{\partial S(x,\rho)}{\partial\rho}=R(x)\frac{(1/2)\mathrm{sh}(2R(x)\rho)-R(x)\rho}{\mathrm{sh}^2(R(x)\rho)}\geqslant 0.$$

(5)根据(2)有 $\partial S(x,\rho)/\partial x=R'(x)\rho\cdot F(x,\rho)$,其中 $F(x,\rho)=\frac{(1/2)\mathrm{sh}(2R(x)\rho)-R(x)\rho}{\mathrm{sh}^2(R(x)\rho)}$.当 ρ

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* 文中 c 均指与 h,ε 无关的常数.

$\geqslant 1$时，$|F(x,\rho)|\leqslant c$，所以$\left|\frac{\partial S(x,\rho)}{\partial x}\right|\leqslant c\rho$，当$\rho\leqslant 1$时

$$\frac{\partial S(x,\rho)}{\partial x}=\frac{R'(x)}{R(x)}[R(x)\rho\coth(R(x)\rho)-R^2(x)\rho^2\cdot \mathrm{sh}^{-2}R(x)\rho]$$

$$=\frac{R'(x)}{R(x)}[S(x,\rho)-1+1-(R(x)\rho)^2\mathrm{sh}^{-2}(R(x)\rho)],$$

根据不等式$|1-z^2\cdot\mathrm{sh}^{-2}z|\leqslant cz^2$和$|S(x,\rho)-1|\leqslant c\rho$，所以$|\partial S(x,\rho)/\partial x|\leqslant c(\rho+\rho^2)\leqslant c\rho$.

(6)考虑到式(2)$\partial S(x,\rho)/\partial\rho\geqslant 0$，所以$S(x,\rho)\geqslant\lim\limits_{\rho\to 0}S(x,\rho)=1$.

(7)$$Q(x_i,\rho)=-\frac{1}{2}\left\{q_i\coth\frac{q_i\rho}{2p_{i+\frac{1}{2}}}-q_{i-1}\coth\frac{q_{i-1}\rho}{2p_{i-\frac{1}{2}}}\right\}+\frac{q_i-q_{i-1}}{2}$$

$$\leqslant-\frac{1}{2}\left\{q_i\coth\frac{q_i\rho}{2p_{i-\frac{1}{2}}}-q_{i-1}\coth\frac{q_{i-1}\rho}{2p_{i-\frac{1}{2}}}\right\}+\frac{q_i-q_{i-1}}{2}$$

$$=\frac{q_{i-1}-q_i}{2}\cdot\frac{\mathrm{sh}\frac{\theta\rho}{2p_{i-\frac{1}{2}}}\left(\mathrm{ch}\frac{\theta\rho}{2p_{i-\frac{1}{2}}}-\mathrm{sh}\frac{\theta\rho}{2p_{i-\frac{1}{2}}}\right)-\frac{\theta\rho}{2p_{i-\frac{1}{2}}}}{\mathrm{sh}^2\frac{\theta\rho}{2p_{i-\frac{1}{2}}}}$$

$$\leqslant 0,\qquad \theta\in(q_i,q_{i-1}).$$

(8)因$\frac{\partial S(x,\rho)}{\partial x}=R'(x)\rho\frac{(1/2)sh(2R(x)\rho)-R(x)\rho}{\mathrm{sh}^2(R(x)\rho)}$，而$\mathrm{sh}^2(R(x)\rho)=2R(x)\rho+t$，这里$|t|\leqslant\frac{h^3\exp(2R(x)\rho)}{\varepsilon(h^2+\varepsilon^2)}$，又$\mathrm{sh}^2(R(x)\rho)\geqslant c\frac{h^2}{h^2+\varepsilon^2}\exp(2R(x)\rho)$. 所以当$h\leqslant\varepsilon$时，$\left|\frac{\partial S(x,\rho)}{\partial x}\right|\leqslant c\frac{h^2}{\varepsilon(h+\varepsilon)}$. 根据引理3，可以证明下列关系式.

引理4　(1)$L^h\exp(-\frac{a^*x_{i-1}}{\varepsilon})\leqslant-\frac{c}{\max(h,\varepsilon)}\exp(-\frac{a^*x_{i-1}}{\varepsilon})$；(2)$L^h(1-x_i)\leqslant-\beta$；(3)$L^hx_i\leqslant c\bar{a}+\varepsilon\bar{p}+\bar{\beta}$；(4)在$[h,1-h]$定义函数：$p_0(x)=x(x+h-2)/(2(1-h))+1$，则$L^hp_0(x_i)\leqslant(c\bar{a}+c\bar{a}h)/(1-h)$，$1\leqslant i\leqslant N-1$；(5)在$[h,1-h]$定义函数：$p_1(x)=x(x-h)/(2(1-h))$，则$L^hp_1(x_i)\leqslant(\bar{\beta}+\bar{p}c+c\bar{a}+c\bar{a}h)/(1-h)$；(6)$L^h1\leqslant-b^*$.

证明　(1)令$w(x)=\exp(-a^*(x-h)/\varepsilon)$，则

$$L^hw(x_i)=\varepsilon\delta(\sigma_ip(x_i)\cdot\delta w(x_i))+D_0(q(x_i)w(x_i))+r(x_i)w(x_i)$$

$$=\frac{\varepsilon}{h}\left\{\sigma_{i+\frac{1}{2}}p_{i+\frac{1}{2}}\frac{w(x_{i+1})-w(x_i)}{h}-\sigma_{i-\frac{1}{2}}p_{i-\frac{1}{2}}\frac{w(x_{i+1})-w(x_i)}{h}\right.$$

$$\left.+\sigma_{i-\frac{1}{2}}p_{i-\frac{1}{2}}\frac{w(x_{i+1})-2w(x_i)+w(x_{i-1})}{h}\right\}$$

$$+\frac{1}{2h}q_{i-1}[w(x_{i+1})-w(x_{i-1})]+\frac{1}{2h}[q_{i+1}-q_{i-1}]w(x_{i+1})+r(x_i)w(x_i)$$

$$\leqslant-c\frac{1}{\max(h,\varepsilon)}w(x_i)+\frac{\varepsilon}{h}[\sigma_{i+\frac{1}{2}}p_{i+\frac{1}{2}}-\sigma_{i-\frac{1}{2}}p_{i-\frac{1}{2}}]\frac{w(x_{i+1})-w(x_i)}{h}$$

$$+\frac{1}{2h}[q_{i+1}-q_{i-1}]w(x_{i+1})+r(x_i)w(x_i),$$

另外还有

$$L^hw(x_i)=\frac{\varepsilon}{h}\left\{\sigma_{i+\frac{1}{2}}p_{i+\frac{1}{2}}\frac{w(x_{i+1})-2w(x_i)+w(x_{i-1})}{h}+\right.$$

$$+\sigma_{i+\frac{1}{2}}p_{i+\frac{1}{2}}\frac{w(x_i)-w(x_{i-1})}{h}-\sigma_{i-\frac{1}{2}}p_{i-\frac{1}{2}}\frac{w(x_i)-w(x_{i-1})}{h}\Big\}$$

$$+\frac{1}{2h}\{q_i[w(x_{i+1})-w(x_{i-1})]+(q_{i+1}-q_i)[w(x_{i+1})-w(x_{i-1})]$$

$$+(q_{i+1}-q_{i-1})w(x_{i-1})\}+r(x_i)w(x_i)$$

$$\leqslant -c\frac{1}{\max(h,\varepsilon)}w(x_i)+\frac{\varepsilon}{h}[\sigma_{i+\frac{1}{2}}p_{i+\frac{1}{2}}-\sigma_{i-\frac{1}{2}}p_{i-\frac{1}{2}}]\frac{w(x_i)-w(x_{i-1})}{h}$$

$$+\frac{1}{2h}(q_{i+1}-q_i)[w(x_{i+1})-w(x_{i-1})]+\frac{1}{2h}[q_{i+1}-q_{i-1}]w(x_{i-1})+r(x_i)w(x_i),$$

由上面两个不等式推出

$$2L^hw(x_i)\leqslant -c\frac{1}{\max(h,\varepsilon)}w(x_i)+\frac{\varepsilon}{h}[\sigma_{i+\frac{1}{2}}p_{i+\frac{1}{2}}-\sigma_{i-\frac{1}{2}}p_{i-\frac{1}{2}}]\frac{w(x_{i+1})-w(x_{i-1})}{h}$$

$$+\frac{1}{2h}(q_{i+1}-q_i)[w(x_{i+1})-w(x_{i-1})]$$

$$+\frac{1}{2h}(q_{i+1}-q_{i-1})[w(x_{i+1})+w(x_{i-1})]+2r(x_i)w(x_i)$$

$$\leqslant -c\frac{1}{\max(h,\varepsilon)}w(x_i)+\frac{1}{h}w(x_i)sh(a^*\rho)\Big\{-q_i\coth\frac{q_i\rho}{2p_{i+\frac{1}{2}}}+q_{i-1}\coth\frac{q_{i-1}\rho}{2p_{i-\frac{1}{2}}}$$

$$-(q_{i+1}-q_i)+(q_{i+1}-q_{i-1})\coth(a^*\rho)\Big\}$$

$$\leqslant -c\frac{1}{\max(h,\varepsilon)}w(x_i),$$

所以 $L^hw(x_i)\leqslant -c\dfrac{1}{\max(h,\varepsilon)}w(x_i)$. 直接计算可推出其余关系式.

根据引理4,容易证得

引理 5　若 $|L^hu_i|\leqslant k\left\{1+\dfrac{1}{\max(h,\varepsilon)}\exp(-\dfrac{a_{i-1}^*}{\varepsilon})\right\}$, $1\leqslant i\leqslant N-1$, $|B_0^hu_0|\leqslant k_0\beta_0\cdot\left\{1+\dfrac{1-\exp(-A^*\rho)}{h}\right\}$和$|B_1^hu_N|\leqslant k_1$都成立,则$|u_i|\leqslant c$, $i=0,1,\cdots,N$. 与文[2]证明方法一样,首先定义式(6),(7)为

$$\begin{cases}L^hV_i=LV(x_i),\\ B_0^hV_0=B_0V(0),\\ B_1^hV_N=B_1V(1),\end{cases}\tag{6}$$

$$\begin{cases}L^hZi=LZ(x_i),\\ B_0^hZ_0=B_0Z(0),\\ B_1^hZ_N=B_1Z(1),\end{cases}\tag{7}$$

那么格式(5)的解 $u_i=\bar{r}V_i+Zi$.

引理6　若 V_i 是方程(6)的解,则$|\bar{r}[V(x_i)-V_i]|\leqslant ch$.

证明　因 $\varepsilon\delta(\sigma_ip(x_i)\delta V(x_i))=F_7+\varepsilon\sigma_{i-\frac{1}{2}}\delta(p_i\delta V(x_i))$,其中 $F_7=\varepsilon p_{i+\frac{1}{2}}\cdot\delta\sigma_i\cdot\delta V(x_{i+\frac{1}{2}})$. 注意到此处 $\sigma_{i-\frac{1}{2}}$与文[2]的 σ_i 相同,所以 $|L^h(V(x_i)-V_i|\leqslant\sum_{i=1}^{7}|Fi|$,其中 $Fi(1\leqslant i\leqslant 6)$与文[2]处 Fi 相同,因此只须估计 F_7. 事实上,

$$|F_7| \leqslant \bar{\alpha}\varepsilon \left|\frac{\partial S(\zeta,\rho)}{\partial x}\right| \cdot \left\{\exp\left(-\frac{q(0)x_{i+\frac{1}{2}}}{p(0)\varepsilon}\right)\operatorname{sh}\frac{q(0)\rho}{2p(0)}\right\}/h$$

$$\leqslant \bar{\alpha}c \exp\left(-\frac{q(0)x_{i+\frac{1}{2}}}{p(0)\varepsilon}\right)\operatorname{sh}\frac{q(0)\rho}{2p(0)}$$

$$\leqslant ch\frac{1}{\max(h,\varepsilon)}\exp\left(-\frac{a^* x_{i-1}}{\varepsilon}\right), \quad \zeta \in (x_{i-\frac{h}{2}}, x_{i+\frac{h}{2}}),$$

$$|L^h(V(x_i) - V_i| \leqslant ch\left\{1 + \frac{1}{\max(h,\varepsilon)}\exp\left(-\frac{a^* x_{i-1}}{\varepsilon}\right)\right\}.$$

边界条件的处理与文〔2〕相同.于是证得$|\bar{r}[V(x_i)-V_i]| \leqslant ch$.

引理7　若z_i是方程(7)的解,则$|z(x_i)-z_i| \leqslant ch$.

证明　因$L^h[z(x_i)-z_i]=G_1+G_2+G_3$,其中$G_1=\varepsilon\delta([\sigma_i-1]p_i\delta z(x_i))$,$G_2$和$G_3$与文〔2〕中的$G_2$、$G_3$相同,由文〔3〕恒等式

$$\delta(g(x)\delta k(x)) = g\left(x+\frac{h}{2}\right)\delta^2 k(x) + \frac{k(x)-k(x-h)}{h^2}\int_{x-\frac{h}{2}}^{x+\frac{h}{2}} g'(t)dt,$$

得

$$|\delta([\sigma_i - 1]p(x_i)\delta z(x_i))|$$

$$\leqslant c|1-\sigma_{i+\frac{1}{2}}| \cdot |\delta^2 z(x_i)| + c\max_{x\in(x_{i-1},x_i)}|z'(x)|\left\{|S(\zeta_1,\rho)-1| + \left|\frac{\partial S(\zeta_2,\rho)}{\partial x}\right|\right\},$$

$$\zeta_1,\zeta_2 \in \left(x_i - \frac{h}{2}, x_i + \frac{h}{2}\right).$$

根据引理3和不等式

$$|\delta^2 z(x_i)| \leqslant ch^{-1}\int_{x_i-h}^{x_i+h}|Z''(x)|dx$$

$$\leqslant c\left\{1 + \frac{1}{\max(h,\varepsilon)}\exp\left(-\frac{a^* x_{i-1}}{\varepsilon}\right)\right\},$$

得

$$|G_1| \leqslant ch\left\{1 + \frac{1}{\max(h,\varepsilon)}\exp\left(-\frac{a^* x_{i-1}}{\varepsilon}\right)\right\},$$

因此$|L^h[z(x_i)-z_i]| \leqslant ch\left\{1+\frac{1}{\max(h,\varepsilon)}\exp\left(-\frac{a^* x_{i-1}}{\varepsilon}\right)\right\}$.边界条件仍跟文〔2〕一样处理,这样就得:$|z(x_i)-z_i| \leqslant ch$,结合引理6和引理7,显然下面定理成立.

定理1　设$u(x_i)$,u_i分别是微分方程(4)和差分格式(5)的解,则$|u(x_i)-u_i| \leqslant ch$.

注　对于第一边值问题

$$\begin{cases} Lu(x) \equiv \varepsilon(p(x)u'(x))' + (q(x)u(x))' + r(x)u(x) = f(x), \quad 0 < x < 1, & \text{(8a)} \\ u(0) = A, u(1) = B, & \text{(8b)} \end{cases}$$

其中$p(x) \geqslant \alpha > 0$,$q(x) \geqslant \beta > 0$,$r(x) \leqslant 0$,$p'(x) \geqslant 0$,$q'(x) \leqslant 0$.考虑守恒型差分格式

$$\begin{cases} L^h u_i \equiv \varepsilon\delta(\sigma_i(\rho)p(x_i)\delta u_i) + D_0(q(x_i)u_i) + r(x_i)u_i = f_i, \quad 1 \leqslant i \leqslant N-1, & \text{(9a)} \\ u_0 = A, u_N = B, & \text{(9b)} \end{cases}$$

其中$\sigma_i(\rho) = \frac{q(x_i-0.5h)\rho}{2p(x_i)}\coth\frac{q(x_i-0.5h)\rho}{2p(x_i)}$,　$\rho = h/\varepsilon$.

定理2　若$u(x_i)$、u_i分别是微分方程(8)和差分格式(9)的解,则

$$|u(x_i)-u_i|\leqslant c\frac{h^2}{h+\varepsilon}+c\frac{h^2}{\varepsilon}\exp\left(-\frac{\alpha^* x_i}{\varepsilon}\right).$$

证明　考虑到引理3式(8)，容易推出：当 $h\leqslant\varepsilon$ 时

$$|F_7|\leqslant c\frac{h^2}{h+\varepsilon}\left\{1+\frac{1}{\max(h,\varepsilon)}\exp\left(-\frac{\alpha^* x_{i-1}}{\varepsilon}\right)\right\},$$

$$|G_4|\leqslant c\frac{h^2}{h+\varepsilon}\left\{1+\frac{1}{\max(h,\varepsilon)}\exp\left(-\frac{\alpha^* x_{i-1}}{\varepsilon}\right)\right\},$$

注意到文〔2〕，不难得出定理2的结论.

3　数值例子

考虑方程

$$\begin{cases}\varepsilon(\sqrt{1+x}u'(x))'+\left(\dfrac{1}{\sqrt{1+x}}u(x)\right)'=\dfrac{1}{2\sqrt{1+x}},\\ u(0)-2u'(0)=1,u(1)+4u'(1)=1,\end{cases}$$

其精确解为 $u(x)=\frac{1+x}{1+\varepsilon}+2k_1\frac{\sqrt{1+x}}{2+\varepsilon}+k_2(1+x)^{-\frac{1}{\varepsilon}}$，这里 $k_1=\left[1-\frac{6}{1+\varepsilon}-\left(1+\frac{1}{1+\varepsilon}\right)/2^{\frac{1}{\varepsilon}}\right](2+\varepsilon)/(4\sqrt{2})$，$k_2=\left[1+\frac{1}{1+\varepsilon}\right]/\left(1-\frac{2}{\varepsilon}\right)$.

采用追赶法，计算结果表明：数值解与精确解十分接近，这与前面的理论分析相符合，表1—2中的误差是指精确解同数值解之差.

表1　$H=0.02$时数值解和误差表

ε	坐标点	数值解	误　差
	0	−0.7317709	−3.489447E−2
	1/50	−0.7490886	−6.53563E−3
10^{-2}	5/50	−0.7408763	−2.03675E−3
	49/50	−0.4948151	−2.703667E−3
	50/50	−0.487383	−2.720833E−3
	0	−0.7433078	−2.433891E−3
10^{-3}	1/50	−0.7607408	−3.49468E−3
	50/50	−0.4955789	−3.422022E−3
	0	−0.7433083	−2.445859E−2
10^{-6}	1/50	−0.7607415	−4.614413E−3
	50/50	−0.495579	−4.420042E−3

表2　$\varepsilon=10^{-3}$时数值解和误差表

H	坐标点	数值解	误差
	0	−0.7190936	−4.855299E−2
0.04	15/25	−0.6273681	−7.617533E−3
	25/25	−0.4910611	−7.939786E−3
	0	−0.7553903	−1.225644E−2
0.01	60/100	−0.6337877	−1.197875E−3
	100/100	−0.4976622	−1.33878E−3

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Conservative Difference Scheme of Conservative Equation with a Small Parammeter

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Abstract This paper deals with conservative equation with a small parammeter. The corresponding conservative difference scheme is proved to be first order uniformly convergent. The uniform convergence can be further improved when the boundary condition degenerates into first boundary condition.

Key words small parammeter, conservative equation, conservative difference scheme, uniform convergence