

异形板塑性计算的研究

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摘 要

本文对几种支承条件的三角形钢筋砼楼板,用塑性分析原理推导出应用简便的计算公式,编制出一套相应的系数计算表格。

一、前 言

钢筋砼规则板如矩形,按塑性计算,现已有较成熟的计算公式和一套相应的计算表格;对不规则板如三角形板和梯形板等,目前尚无现成公式和表格。随着我国高层建筑(高层建筑的特点之一是平面形状多变)的发展,不规则板的比例显著增加。因此,在设计(高层建筑项目中常遇不规则楼板,却无现成的计算公式很感不便。为此,笔者仅就三角形板按塑性分析的要求,对几种支承条件分别分析,推导出形式简单的计算公式,编制了一套相应的计算表格。供设计时查用。使用简便。这对塑性分析的应用和推广是有利的。

二、用塑性理论分析板的内力

三角形板与矩形板一样,是一种高次超静定结构,要精确计算其极限荷载和内力值是很困难的。有几种常用的近似求极限荷载的方法,如机动法、极限平衡法等,是大家所熟悉的。现用文[7]的分量矢量法,求解极限荷载与板的屈服弯矩之间的关系。该法的基本原理是板在破坏时,整块板为塑性铰线分割成为若干块刚性区域,将内力、转角、转动轴视为矢量分析塑性铰线上的屈服弯矩在变位时消耗的总能量,并使之与极限荷载所做的总功相平衡,从而求出极限荷载或屈服弯矩。实际上,用屈服线分析求得的极限荷载比破坏荷载的实际值低得多^[8],故其可靠指标较高。根据这种原理,就下列几种支承条件的三角形板,推导出相应的计算公式。

1. 两边简支一边自由的三角形板

如图1所示, ab 和 ac 两边为简支边, bc 为自由边,板用各向同性均匀配筋,即 $m_1 = m_2 =$

本文1987年6月2日收到。

$m_3 = m_4 = m$, 板上作用均布荷载 q , 要确定当板达到塑性破坏时, 极限荷载 q 与板所能承担的屈服弯矩 m 之间的关系.

假设在 d 点产生单位位移 (往下), 确定极限荷载 q 所做的外功 W 和屈服弯矩 m 消耗的内能 U .

(1) 外功: 极限荷载所做的外功等于板变形的体积 (三角锥) 乘以 q

$$W = \frac{1}{3}(q)\left(\frac{1}{2}L_1L_2\sin\beta\right) = \frac{1}{6}qL_1L_2\sin\beta \quad (1)$$

(2) 消耗的内能: 用分量矢量法确定在变位时屈服弯矩消耗的能量

$$U = U_A + U_B = m_1\left[(ad)\cos(\beta-\alpha)\frac{1}{(ad)\sin(\beta-\alpha)}\right] + m_2[0] + m_3\left[(ad)\cos\alpha\frac{1}{(ad)\sin\alpha}\right] + m_4[0] = m_1\text{ctg}(\beta-\alpha) + m_3\text{ctg}\alpha \quad (2)$$

由式(1)、(2)得

$$\frac{m}{q} = \frac{1}{6}L_1L_2\sin\alpha\sin(\beta-\alpha) \quad (3)$$

为确定塑性铰线最不利位置,

$$\frac{d(m/q)}{d\alpha} = -\sin\alpha\cos(\beta-\alpha) + \sin(\beta-\alpha)\cos\alpha = 0$$

$$\sin[(\beta-\alpha)-\alpha] = 0 \quad \alpha = \frac{\beta}{2}$$

将 α 值代入式(3), 即得

$$m = \frac{1}{6}L_1L_2q\sin\frac{\beta}{2} \quad (N\cdot m)$$

2. 两边固定一边自由的三角形板

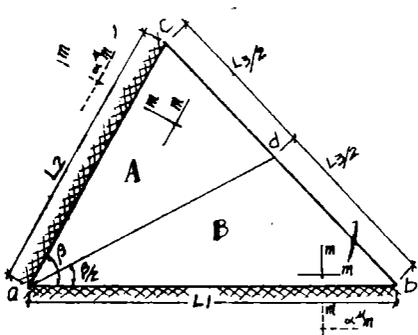


图 2

如图 2 所示, 板跨中为各向同性配筋, 且各与固定边正交, 垂直于支座边的支座钢筋为跨中的 α 倍. 同理可求得正塑性铰线的最不利位置是 β 角的等分线. 负塑性铰线产生在支承边.

同理得

$$w = \frac{1}{6}L_1L_2q\sin\beta$$

$$U = U_A + U_B = m\left[(ad)\cos\frac{\beta}{2}\frac{1}{(ad)\sin\frac{\beta}{2}}\right] + \alpha^2m\left[(ac)\frac{1}{(ad)\sin\frac{\beta}{2}}\right] +$$

$$+ m \left[(ad) \cos \frac{\beta}{2} \cdot \frac{1}{(ad) \sin \frac{\beta}{2}} \right] + \alpha^{\mu} m \left[(ad) \frac{1}{(ad) \sin \frac{\beta}{2}} \right]$$

化简后得

$$U = 2n \operatorname{ctg} \frac{\beta}{2} + \frac{2m}{\sin \beta} \alpha^{\mu} (1+n)$$

即得

$$\frac{1}{6} L_1 L_2 \sin \beta = 2m \left[\operatorname{ctg} \frac{\beta}{2} + \frac{1}{\sin \beta} 2^{\mu} (1+n) \right]$$

$$m = \frac{1}{12} L_1 L_2 q \sin^2 \beta / \left[\operatorname{ctg} \frac{\beta}{2} \sin \beta + 2^{\mu} (1+n) \right].$$

令 $v = \frac{\sin^2 \beta}{\sin \beta \operatorname{ctg} \frac{\beta}{2} + \alpha^{\mu} (1+n)}$ (系数) 由表 1 可查获。所以

$$m = \frac{1}{12} L_1 L_2 v q \quad (\text{Nm}) \tag{4}$$

表 1 α^{μ} 分别为 1.0、1.5 和 2.0 时的 v 值

$\beta(^{\circ})$	n										
	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
$\alpha^{\mu} = 1.0$											
45	.1349	.1313	.1280	.1248	.1217	.1188	.1161	.1134	.1109	.1085	.1062
50	.1611	.1568	.1527	.1488	.1451	.1416	.1383	.1351	.1321	.1292	.1264
55	.1878	.1827	.1778	.1732	.1689	.1647	.1608	.1570	.1532	.1500	.1467
60	.2143	.2083	.2027	.1974	.1923	.1875	.1829	.1786	.1744	.1704	.1667
65	.2400	.2332	.2267	.2206	.2149	.2094	.2042	.1992	.1945	.1900	.1857
70	.2642	.2565	.2493	.2424	.2360	.2298	.2240	.2185	.2132	.2082	.2034
75	.2863	.2778	.2697	.2622	.2550	.2482	.2418	.2357	.2299	.2244	.2191
80	.3056	.2963	.2875	.2792	.2714	.2640	.2570	.2504	.2441	.2381	.2324
85	.3215	.3114	.3019	.2930	.2846	.2766	.2691	.2620	.2553	.2489	.2428
90	.3333	.3226	.3125	.3030	.2941	.2857	.2778	.2703	.2632	.2564	.2500
95	.3407	.3294	.3188	.3089	.2996	.2908	.2825	.2747	.2673	.2603	.2536
100	.3431	.3314	.3205	.3102	.3006	.2916	.2831	.2750	.2674	.2602	.2535
105	.3404	.3284	.3172	.3068	.2970	.2879	.2793	.2711	.2635	.2562	.2494
110	.3322	.3203	.3090	.2985	.2888	.2796	.2710	.2630	.2554	.2482	.2414
115	.3187	.3066	.2958	.2855	.2759	.2669	.2585	.2506	.2432	.2362	.2296
120	.3000	.2885	.2778	.2679	.2586	.2500	.2419	.2344	.2273	.2206	.2143
125	.2766	.2656	.2555	.2461	.2374	.2293	.2217	.2146	.2080	.2017	.1958
130	.2490	.2388	.2295	.2209	.2129	.2054	.1985	.1920	.1859	.1802	.1748

表 1 (续)

$\alpha^* = 1.5$											
45	.1062	.1029	.0999	.0969	.0942	.0916	.0892	.0868	.0845	.0825	.0805
50	.1264	.1224	.1187	.1152	.1119	.1088	.1059	.1031	.1004	.0979	.0955
55	.1467	.1420	.1377	.1336	.1297	.1260	.1226	.1193	.1162	.1133	.1105
60	.1667	.1613	.1562	.1515	.1471	.1429	.1389	.1351	.1316	.1282	.1250
65	.1857	.1796	.1739	.1686	.1635	.1586	.1543	.1501	.1461	.1423	.1387
70	.2034	.1966	.1902	.1843	.1787	.1734	.1684	.1638	.1593	.1551	.1511
75	.2191	.2116	.2047	.1981	.1920	.1863	.1809	.1757	.1709	.1663	.1620
80	.2324	.2243	.2168	.2098	.2032	.1970	.1911	.1857	.1805	.1756	.1709
85	.2428	.2342	.2262	.2187	.2117	.2052	.1990	.1932	.1877	.1825	.1776
90	.2500	.2410	.2326	.2247	.2174	.2105	.2041	.1980	.1923	.1869	.1818
95	.2536	.2443	.2356	.2275	.2199	.2128	.2062	.2000	.1941	.1886	.1833
100	.2535	.2439	.2350	.2268	.2191	.2119	.2052	.1989	.1930	.1874	.1821
105	.2494	.2398	.2309	.2226	.2149	.2077	.2010	.1947	.1888	.1833	.1780
110	.2414	.2319	.2231	.2150	.2074	.2003	.1937	.1876	.1818	.1763	.1712
115	.2296	.2204	.2119	.2040	.1966	.1898	.1835	.1775	.1719	.1667	.1618
120	.2143	.2055	.1974	.1899	.1829	.1765	.1705	.1648	.1596	.1546	.1500
125	.1958	.1876	.1801	.1731	.1667	.1607	.1551	.1499	.1450	.1405	.1362
130	.1748	.1673	.1605	.1541	.1483	.1429	.1379	.1332	.1288	.1247	.1208
$\alpha^* = 2.0$											
45	.0878	.0846	.0819	.0793	.0768	.0745	.0724	.0703	.0684	.0666	.0649
50	.1040	.1004	.0971	.0940	.0911	.0883	.0858	.0833	.0810	.0788	.0768
55	.1204	.1162	.1123	.1087	.1053	.1021	.0991	.0962	.0938	.0910	.0886
60	.1364	.1316	.1271	.1229	.1190	.1154	.1119	.1087	.1056	.1027	.1000
65	.1515	.1461	.1411	.1364	.1320	.1279	.1240	.1204	.1170	.1137	.1107
70	.1653	.1593	.1538	.1486	.1438	.1392	.1350	.1310	.1272	.1236	.1203
75	.1774	.1709	.1649	.1592	.1540	.1491	.1445	.1401	.1360	.1322	.1285
80	.1875	.1805	.1740	.1680	.1624	.1571	.1522	.1475	.1432	.1391	.1352
85	.1951	.1877	.1809	.1745	.1686	.1630	.1578	.1530	.1484	.1441	.1400
90	.2000	.1923	.1852	.1786	.1724	.1667	.1613	.1562	.1515	.1471	.1429
95	.2020	.1941	.1868	.1800	.1737	.1678	.1623	.1572	.1524	.1478	.1436
100	.2009	.1930	.1856	.1787	.1724	.1665	.1609	.1558	.1509	.1464	.1421
105	.1968	.1888	.1815	.1747	.1684	.1625	.1570	.1519	.1471	.1426	.1384
110	.1896	.1818	.1746	.1679	.1618	.1561	.1507	.1458	.1411	.1367	.1326
115	.1795	.1719	.1650	.1587	.1526	.1473	.1422	.1374	.1330	.1288	.1249
120	.1667	.1596	.1531	.1471	.1415	.1364	.1316	.1271	.1230	.1191	.1154
125	.1516	.1450	.1390	.1335	.1284	.1237	.1193	.1152	.1114	.1078	.1044
130	.1347	.1288	.1234	.1184	.1138	.1096	.1056	.1019	.0985	.0953	.0923

3. 两边固定一边简支的三角形板 (I 型)

bc 为简支边, 其余情况与 2 相同。

当板破坏时, ad 仍为 β 角的等分线。但因 bc 为简支边, 塑性铰线 bd 和 cd 的最不利位置尚需进一步探讨。负塑性铰线产生在固定支承边^[8]。

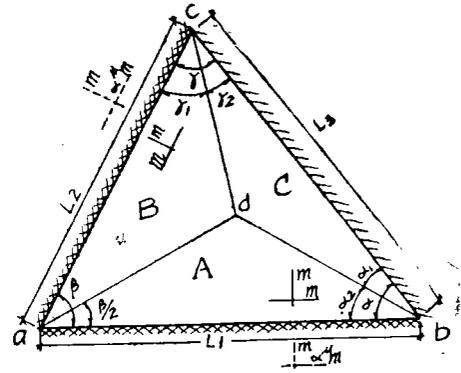


图 3

同理

$$w = \frac{1}{6} L_1 L_2 q \sin \beta$$

$$U = U_A + U_B + U_C$$

$$= m \left[(ad) \cos \frac{\beta}{2} \frac{1}{(ad) \sin \frac{\beta}{2}} \right]$$

$$+ m \left[(bd) \cos \alpha_2 \frac{1}{(ad) \sin \frac{\beta}{2}} \right] + \alpha^\mu m \left[(ab) \frac{1}{(ad) \sin \frac{\beta}{2}} \right]$$

$$+ m \left[(ad) \cos \frac{\beta}{2} \frac{1}{(ad) \sin \frac{\beta}{2}} \right] + m \left[(ac) \cos \alpha_2 \frac{1}{(ad) \sin \frac{\beta}{2}} \right]$$

$$+ \alpha^\mu m \left[(ac) \frac{1}{(ad) \sin \frac{\beta}{2}} \right] + m \left[(bc \cos \alpha) \left(\frac{1}{db \sin \alpha_1} \cos \alpha \right) \right]$$

$$+ (bc \sin \alpha) \left(\frac{1}{db \sin \alpha_1} \sin \alpha \right) \Big] + m \left[(bc \cos \gamma) \left(\frac{1}{db \sin \alpha_1} \cos \gamma \right) \right]$$

$$+ (bc \sin \gamma) \left(\frac{1}{db \sin \alpha_1} \sin \gamma \right) \Big]$$

化简后得

$$U = 2m(1 + \alpha^\mu) \operatorname{ctg} \frac{\beta}{2} + m(1 + \alpha^\mu) (\operatorname{ctg} \alpha_2 + \operatorname{ctg} \gamma_2) + 2m [\operatorname{ctg} (\alpha - \alpha_2) + \operatorname{ctg} (\gamma - \gamma_2)]$$

令 $w = U$, 得

$$\frac{1}{6} L_1 L_2 q \sin \beta = m \left\{ 2(1 + \alpha^\mu) \operatorname{ctg} \frac{\beta}{2} + (1 + \alpha^\mu) (\operatorname{ctg} \alpha_2 + \operatorname{ctg} \gamma_2) + 2[\operatorname{ctg} (\alpha - \alpha_2) + \operatorname{ctg} (\gamma - \gamma_2)] \right\}$$

$$\frac{q}{m} = \frac{6}{L_1 L_2 \sin \beta} \left\{ 2(1 + \alpha^\mu) \operatorname{ctg} \frac{\beta}{2} + (1 + \alpha^\mu) (\operatorname{ctg} \alpha_2 + \operatorname{ctg} \gamma_2) + 2[\operatorname{ctg} (\alpha - \alpha_2) + \operatorname{ctg} (\gamma - \gamma_2)] \right\} \quad (5)$$

$$\frac{\partial (q/m)}{\partial \alpha_2} = 0 \quad \frac{\partial (q/m)}{\partial \gamma_2} = 0$$

$$\frac{\partial (q/m)}{\partial \alpha_2} = \frac{6}{L_1 L_2 \sin \beta} [-(1 + \alpha^\mu) \operatorname{csc}^2 \alpha_2 - 2 \operatorname{csc}^2 (\alpha - \alpha_2) (-1)] = 0$$

$$(1 + \alpha^\mu) \frac{1}{\sin^2 \alpha_2} = \frac{2}{[\sin (\alpha - \alpha_2)]^2} \quad \sin \alpha_1 = \sqrt{\frac{2}{1 + \alpha^\mu}} \sin \alpha_2$$

$$\alpha_1 = \sqrt{\frac{2}{1+\alpha^\mu}} \alpha_2 \quad \alpha_1 + \alpha_2 = \alpha \quad \alpha_2 = \left(\frac{1}{1 + \sqrt{\frac{2}{1+\alpha^\mu}}} \alpha \right) \quad \frac{\alpha(q/m)}{\alpha\gamma_2} = 0$$

同理可求得

$$\gamma_2 = \left(\frac{1}{1 + \sqrt{\frac{2}{1+\alpha^\mu}}} \right) \gamma$$

当 $\alpha^\mu = 1.0$ 时

$$\alpha_1 = \alpha_2, \quad \gamma_1 = \gamma_2$$

这说明板若是分别按与两固定边正交的方式配筋，并延伸到简支边，当支座与板中配筋率相同时($\alpha^\mu = 1.0$)，其塑性铰线 bd 和 cd 的最不利位置是各自夹角的等分线。

将 α_2, γ_2 值代入式5，化简后得

$$\begin{aligned} \frac{q}{m} = & \frac{6}{L_1 L_2 \sin \beta} \left\{ 2(1+\alpha^\mu) \operatorname{ctg} \frac{\beta}{2} + (1+\alpha^\mu) \left[\operatorname{ctg} \left(\frac{1}{1 + \sqrt{\frac{2}{1+\alpha^\mu}}} \alpha \right) \right. \right. \\ & \left. \left. + \operatorname{ctg} \left(\frac{1}{1 + \sqrt{\frac{2}{1+\alpha^\mu}}} \gamma \right) \right] + 2 \left[\operatorname{ctg} \left(\frac{\sqrt{\frac{2}{1+\alpha^\mu}}}{1 + \sqrt{\frac{2}{1+\alpha^\mu}}} \alpha \right) + \operatorname{ctg} \left(\frac{\sqrt{\frac{2}{1+\alpha^\mu}}}{1 + \sqrt{\frac{2}{1+\alpha^\mu}}} \gamma \right) \right] \right\} \end{aligned}$$

令

$$\begin{aligned} \omega_1 = & \sin \beta \left\{ 2(1+\alpha^\mu) \operatorname{ctg} \frac{\beta}{2} + (1+\alpha^\mu) \left[\operatorname{ctg} \left(\frac{1}{1 + \sqrt{\frac{2}{1+\alpha^\mu}}} \alpha \right) \right. \right. \\ & \left. \left. + \operatorname{ctg} \left(\frac{1}{1 + \sqrt{\frac{2}{1+\alpha^\mu}}} \gamma \right) \right] + 2 \left[\operatorname{ctg} \left(\frac{\sqrt{\frac{2}{1+\alpha^\mu}}}{1 + \sqrt{\frac{2}{1+\alpha^\mu}}} \alpha \right) + \operatorname{ctg} \left(\frac{\sqrt{\frac{2}{1+\alpha^\mu}}}{1 + \sqrt{\frac{2}{1+\alpha^\mu}}} \gamma \right) \right] \right\} \end{aligned}$$

ω_1 是与 $\beta, \alpha, \gamma, \alpha^\mu$ 有关的系数(表2)则得

$$m = \frac{L_1 L_2 \omega_1}{6} q \quad (N \cdot m) \quad (6)$$

4. 两边固定一边简支的三角形板(II型)

II型板在靠近简支边附近采用与板边正交方式进行配筋，其余与I型相同(图4)。因此，仅“C”刚性块的能量消耗有所不同，即

$$\begin{aligned} U_c = & m \left[(db) \cos \alpha_1 \frac{1}{db \sin \beta} + 0 \right] \\ & + m \left[(dc) \cos \gamma_1 \frac{1}{db \sin \beta} + 0 \right] \\ = & m (\operatorname{ctg} \alpha_1 + \operatorname{ctg} \gamma_1) \end{aligned}$$

$$U = 2m(1+\alpha^\mu) \operatorname{ctg} \frac{\beta}{2} + m(1+\alpha^\mu) (\operatorname{ctg} \alpha_2 + \operatorname{ctg} \gamma_2) + m (\operatorname{ctg} \alpha_1 + \operatorname{ctg} \gamma_1)$$

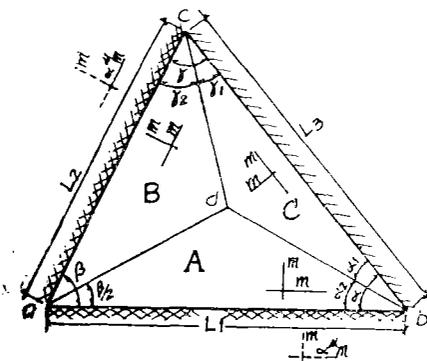


图4

$$\frac{q}{m} = \frac{6}{L_1 L_2 \sin \beta} \left[2(1 + \alpha^\mu) \operatorname{ctg} \frac{\beta}{2} + (1 + \alpha^\mu) (\operatorname{ctg} \alpha_2 + \operatorname{ctg} \gamma_2) + (\operatorname{ctg} \alpha_2 + \operatorname{ctg} \gamma_1) \right] \quad (7a)$$

通过求偏导数, 同样可求得

$$\alpha_2 = \frac{1}{1 + \sqrt{\frac{1}{1 + \alpha^\mu}}} \alpha \quad \gamma_2 = \frac{1}{1 + \sqrt{\frac{1}{1 + \alpha^\mu}}} \gamma \quad (7b)$$

当 $\alpha^\mu = 1.0$ 时, $\alpha_2 = 0.586\alpha$ $\gamma_2 = 0.586\gamma$. 这说明塑性铰线 bd 和 cd 往简支边靠拢.

将式(7b)代入式(7a), 整理后得

$$\begin{aligned} \frac{q}{m} = \frac{6}{L_1 L_2 \sin \beta} \left\{ 2(1 + \alpha^\mu) \operatorname{ctg} \frac{\beta}{2} + (1 + \alpha^\mu) \left[\operatorname{ctg} \left(\frac{\sqrt{1 + \alpha^\mu}}{\sqrt{1 + \alpha^\mu} + 1} \alpha \right) \right. \right. \\ \left. \left. + \operatorname{ctg} \left(\frac{\sqrt{1 + \alpha^\mu}}{\sqrt{1 + \alpha^\mu} + 1} \gamma \right) \right] + \left[\operatorname{ctg} \left(\frac{1}{\sqrt{1 + \alpha^\mu} + 1} \alpha \right) \right. \right. \\ \left. \left. + \operatorname{ctg} \left(\frac{1}{\sqrt{1 + \alpha^\mu} + 1} \gamma \right) \right] \right\} \end{aligned}$$

令

$$\begin{aligned} \omega_2 = \sin \beta / \left\{ 2(1 + \alpha^\mu) \operatorname{ctg} \frac{\beta}{2} + (1 + \alpha^\mu) \left[\operatorname{ctg} \left(\frac{\sqrt{1 + \alpha^\mu}}{\sqrt{1 + \alpha^\mu} + 1} \alpha \right) \right. \right. \\ \left. \left. + \operatorname{ctg} \left(\frac{\sqrt{1 + \alpha^\mu}}{\sqrt{1 + \alpha^\mu} + 1} \gamma \right) \right] + \left[\operatorname{ctg} \left(\frac{1}{\sqrt{1 + \alpha^\mu} + 2} \alpha \right) \right. \right. \\ \left. \left. + \operatorname{ctg} \left(\frac{1}{\sqrt{1 + \alpha^\mu}} \gamma \right) \right] \right\} \end{aligned}$$

ω_2 是与 β 、 α 、 γ 、 α^μ 有关的屈服弯矩系数(表3)。于是

$$m = \frac{L_1 L_2 \omega_2}{6} q \quad (N \cdot m) \quad (8)$$

必须说明在同等条件下, 采用 II 型配筋方式布筋, 其屈服弯矩系数 ω_2 比 ω_1 增大 21.4%, 其配筋也必同样增加, 因此较不经济, 而且分别从两固定边正交钢筋延伸到简支边, 施工较为方便。若在简支边附近的板中另搞一套正交于自身的方式布筋(II型), 则施工不便, 且必浪费材料, 在一般情况不宜采用。

三、结束语

本文推导的各种公式和计算系数表, 其计算精度能为实际工程的要求所接受, 在设计工作中的实际应用极为方便。原按 JCSS 采用的 R-F 法对板进行概率设计的部分, 一则篇幅所限, 二则有个别问题尚须进一步探讨。因此, 这一部分内容拟另外成文。

表 2 $\nu^{\#}$ 分别为 1.0、1.5 和 2.0 时的 ω_1 值

$\alpha(^{\circ})$	$\beta(^{\circ})$																	
	45	50	55	60	65	70	75	80	85	90	95	100	105	110	115	120	125	130
$\alpha^{\#} = 1.0$																		
25	.0232	.0258	.0282	.0302	.0320	.0334	.0345	.0351	.0354	.0353	.0348	.0339	.0325	.0308	.0287	.0262	.0234	.0202
30	.0256	.0285	.0312	.0335	.0354	.0370	.0381	.0388	.0390	.0387	.0379	.0367	.0349	.0327	.0300	.0269	.0234	
35	.0275	.0307	.0336	.0361	.0382	.0398	.0409	.0419	.0415	.0410	.0400	.0383	.0361	.0334	.0300	.0262		
40	.0291	.0325	.0356	.0382	.0403	.0419	.0430	.0434	.0432	.0424	.0410	.0389	.0361	.0327	.0287			
45	.0303	.0339	.0371	.0397	.0419	.0434	.0443	.0445	.0441	.0429	.0410	.0383	.0349	.0308				
50	.0313	.0349	.0381	.0408	.0429	.0443	.0450	.0449	.0441	.0424	.0400	.0367	.0325					
55	.0320	.0357	.0389	.0415	.0434	.0446	.0450	.0445	.0432	.0410	.0379	.0339						
60	.0324	.0361	.0392	.0417	.0434	.0443	.0442	.0433	.0415	.0387	.0348							
65	.0327	.0362	.0392	.0415	.0429	.0434	.0430	.0415	.0390	.0353								
70	.0327	.0361	.0389	.0408	.0419	.0418	.0409	.0388	.0354									
75	.0324	.0357	.0381	.0397	.0403	.0398	.0381	.0351										
80	.0320	.0349	.0371	.0382	.0381	.0370	.0344											
85	.0313	.0339	.0356	.0361	.0354	.0334												
90	.0303	.0325	.0336	.0335	.0320													

表 2 (续)

$\alpha(^{\circ})$	$\beta(^{\circ})$																	
	45	50	55	60	65	70	75	80	85	90	95	100	105	110	115	120	125	130
$\alpha^* = 1.5$																		
25	.0200	.0223	.0244	.0263	.0278	.0291	.0301	.0307	.0310	.0309	.0306	.0298	.0287	.0272	.0254	.0232	.0207	.0179
30	.0219	.0245	.0269	.0290	.0308	.0322	.0332	.0338	.0341	.0339	.0333	.0322	.0308	.0289	.0265	.0238	.0207	
35	.0235	.0264	.0289	.0312	.0331	.0345	.0356	.0362	.0363	.0359	.0351	.0337	.0318	.0294	.0265	.0233		
40	.0248	.0278	.0306	.0329	.0348	.0363	.0373	.0378	.0377	.0371	.0359	.0342	.0318	.0289	.0254			
45	.0258	.0290	.0318	.0342	.0361	.0376	.0385	.0388	.0385	.0375	.0359	.0337	.0308	.0272				
50	.0266	.0298	.0327	.0351	.0370	.0383	.0390	.0391	.0385	.0371	.0351	.0322						
55	.0272	.0304	.0333	.0356	.0374	.0386	.0390	.0388	.0378	.0359	.0333	.0298						
60	.0275	.0308	.0336	.0358	.0374	.0383	.0385	.0378	.0363	.0339	.0306							
65	.0277	.0309	.0336	.0356	.0370	.0376	.0373	.0362	.0341	.0310								
70	.0277	.0308	.0333	.0351	.0361	.0363	.0356	.0338	.0310									
75	.0275	.0304	.0321	.0342	.0348	.0345	.0332	.0307										
80	.0272	.0298	.0318	.0329	.0331	.0322	.0301											
85	.0266	.0290	.0306	.0312	.0308	.0291												
90	.0258	.0278	.0289	.0290	.0278													

表 2 (续)

$\alpha(^{\circ})$	$\beta(^{\circ})$																	
	45	50	55	60	65	70	75	80	85	90	95	100	105	110	115	120	125	130
	$\alpha^{\mu} = 2.0$																	
25	.0176	.0197	.0216	.0233	.0247	.0259	.0268	.0274	.0278	.0277	.0274	.0267	.0258	.0244	.0228	.0209	.0187	.0162
30	.0193	.0216	.0238	.0257	.0273	.0286	.0295	.0302	.0304	.0303	.0298	.0289	.0276	.0259	.0239	.0214	.0187	
35	.0206	.0232	.0255	.0275	.0293	.0306	.0316	.0322	.0324	.0321	.0314	.0302	.0285	.0264	.0239	.0209		
40	.0217	.0244	.0269	.0290	.0308	.0322	.0331	.0336	.0335	.0331	.0321	.0306	.0285	.0259	.0228			
45	.0226	.0254	.0279	.0301	.0319	.0333	.0341	.0345	.0343	.0335	.0321	.0302	.0276	.0244				
50	.0232	.0261	.0287	.0309	.0327	.0339	.0346	.0348	.0343	.0331	.0314	.0289	.0258					
55	.0237	.0266	.0292	.0314	.0330	.0341	.0346	.0345	.0336	.0321	.0298	.0267						
60	.0240	.0269	.0295	.0315	.0331	.0339	.0341	.0336	.0324	.0303	.0274							
65	.0242	.0270	.0295	.0314	.0327	.0333	.0331	.0322	.0304	.0277								
70	.0241	.0269	.0292	.0309	.0319	.0322	.0316	.0302	.0278									
75	.0240	.0266	.0287	.0301	.0308	.0306	.0295	.0274										
80	.0237	.0261	.0279	.0290	.0293	.0286	.0268											
85	.0232	.0254	.0269	.0275	.0273	.0259												
90	.0226	.0244	.0255	.0257	.0247													

表 3 α^* 分别为 1.0、1.5 和 2.0 时的 ω_2 值

α^*	β (°)																	
	45	50	55	60	65	70	75	80	85	90	95	100	105	110	115	120	125	130
	$\alpha^* = 1.0$																	
25	.0286	.0321	.0354	.0382	.0407	.0428	.0444	.0455	.0461	.0462	.0457	.0447	.0431	.0410	.0383	.0351	.0314	.0272
30	.0312	.0352	.0388	.0420	.0448	.0470	.0488	.0499	.0505	.0504	.0496	.0482	.0462	.0434	.0401	.0361	.0314	
35	.0333	.0376	.0415	.0450	.0479	.0503	.0521	.0533	.0536	.0533	.0522	.0503	.0477	.0443	.0401	.0351		
40	.0350	.0395	.0436	.0473	.0504	.0528	.0545	.0555	.0557	.0550	.0534	.0510	.0477	.0434	.0383			
45	.0363	.0410	.0453	.0490	.0521	.0544	.0561	.0568	.0567	.0555	.0534	.0503	.0462	.0410				
50	.0373	.0421	.0465	.0502	.0533	.0555	.0569	.0573	.0567	.0550	.0522	.0482	.0431					
55	.0380	.0429	.0472	.0509	.0538	.0559	.0569	.0568	.0557	.0535	.0496	.0447						
60	.0384	.0433	.0476	.0512	.0538	.0555	.0561	.0555	.0536	.0504	.0457							
65	.0387	.0435	.0476	.0509	.0533	.0545	.0544	.0532	.0505	.0462								
70	.0386	.0433	.0472	.0502	.0521	.0528	.0521	.0499	.0461									
75	.0384	.0429	.0465	.0490	.0504	.0503	.0488	.0455										
80	.0380	.0421	.0453	.0473	.0479	.0470	.0444											
85	.0373	.0410	.0436	.0450	.0448	.0428												
90	.0363	.0395	.0415	.0420	.0407													

表 3 (续)

		$\beta^{(*)}$																	
		45	50	55	60	65	70	75	80	85	90	95	100	105	110	115	120	125	130
25	.0242	.0273	.0301	.0326	.0348	.0367	.0381	.0391	.0397	.0398	.0395	.0386	.0373	.0355	.0332	.0305	.0273	.0237	
30	.0264	.0298	.0329	.0357	.0382	.0402	.0418	.0428	.0434	.0433	.0428	.0416	.0399	.0376	.0347	.0313	.0272		
35	.0281	.0318	.0352	.0382	.0408	.0430	.0446	.0456	.0460	.0458	.0450	.0434	.0412	.0383	.0347	.0305			
40	.0294	.0333	.0369	.0401	.0428	.0450	.0466	.0475	.0478	.0473	.0460	.0440	.0412	.0376	.0332				
45	.0304	.0345	.0382	.0415	.0443	.0464	.0479	.0486	.0485	.0477	.0460	.0434	.0399	.0355					
50	.0312	.0354	.0392	.0425	.0452	.0473	.0485	.0490	.0486	.0473	.0450	.0416	.0373						
55	.0318	.0360	.0398	.0431	.0457	.0475	.0485	.0486	.0478	.0458	.0428	.0386							
60	.0322	.0368	.0401	.0433	.0457	.0473	.0479	.0475	.0460	.0434	.0395								
65	.0324	.0325	.0400	.0431	.0452	.0464	.0466	.0456	.0434	.0398									
70	.0324	.0364	.0398	.0425	.0443	.0450	.0446	.0428	.0397										
75	.0322	.0360	.0392	.0415	.0428	.0430	.0418	.0391											
80	.0318	.0354	.0382	.0401	.0408	.0402	.0381												
85	.0312	.0345	.0369	.0382	.0381	.0367													
90	.0304	.0333	.0352	.0357	.0348														

 $\rho^* = 1.5$

表 3 (续)

$\alpha(^{\circ})$	$\beta(^{\circ})$																	
	45	50	55	60	65	70	75	80	85	90	95	100	105	110	115	120	125	130
	$\alpha^{\#} = 2.0$																	
25	.0211	.0238	.0263	.0286	.0305	.0322	.0335	.0344	.0350	.0351	.0348	.0341	.0330	.0315	.0295	.0271	.0243	.0211
30	.0229	.0259	.0287	.0312	.0334	.0352	.0367	.0376	.0382	.0381	.0378	.0368	.0353	.0333	.0308	.0278	.0243	
35	.0242	.0276	.0306	.0333	.0357	.0376	.0391	.0400	.0405	.0403	.0396	.0383	.0364	.0330	.0308	.0271		
40	.0254	.0289	.0321	.0349	.0374	.0394	.0408	.0417	.0420	.0416	.0406	.0388	.0362	.0333	.0295			
45	.0263	.0299	.0332	.0361	.0386	.0406	.0419	.0427	.0426	.0420	.0406	.0383	.0353	.0315				
50	.0270	.0307	.0340	.0370	.0394	.0413	.0425	.0430	.0427	.0416	.0396	.0368	.0330					
55	.0275	.0312	.0345	.0375	.0398	.0415	.0425	.0427	.0420	.0403	.0378	.0341						
60	.0278	.0315	.0348	.0376	.0396	.0413	.0419	.0417	.0405	.0382	.0348							
65	.0279	.0316	.0349	.0375	.0394	.0406	.0404	.0400	.0382	.0351								
70	.0278	.0315	.0345	.0370	.0386	.0394	.0391	.0376	.0350									
75	.0277	.0312	.0340	.0361	.0374	.0376	.0367	.0344										
80	.0275	.0307	.0332	.0349	.0357	.0352	.0335											
85	.0270	.0299	.0321	.0333	.0334	.0322												
90	.0263	.0289	.0306	.0312	.0305													

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Study on the Plastic Calculation of Abnormal Slab

Chen Ruilin

Abstract

By applying the principle of plastic analysis, a simple formula was derived for calculating triangular reinforced concrete floor slab under several supporting conditions, and a set of charts were drawn up for calculating corresponding coefficients. This may be helpful in practical use.